

Mathematica 11.3 Integration Test Results

Test results for the 46 problems in "4.5.10 $(c+dx)^m (a+b \sec(e+fx))^n$ "

Problem 6: Result more than twice size of optimal antiderivative.

$$\int (c + dx)^3 (a + a \operatorname{Sec}[e + fx])^2 dx$$

Optimal (type 4, 371 leaves, 17 steps):

$$\begin{aligned} & -\frac{i a^2 (c + dx)^3}{f} + \frac{a^2 (c + dx)^4}{4 d} - \frac{4 i a^2 (c + dx)^3 \operatorname{ArcTan}[e^{i (e+fx)}]}{f} + \\ & \frac{3 a^2 d (c + dx)^2 \operatorname{Log}[1 + e^{2 i (e+fx)}]}{f^2} + \frac{6 i a^2 d (c + dx)^2 \operatorname{PolyLog}[2, -i e^{i (e+fx)}]}{f^2} - \\ & \frac{6 i a^2 d (c + dx)^2 \operatorname{PolyLog}[2, i e^{i (e+fx)}]}{f^2} - \frac{3 i a^2 d^2 (c + dx) \operatorname{PolyLog}[2, -e^{2 i (e+fx)}]}{f^3} - \\ & \frac{12 a^2 d^2 (c + dx) \operatorname{PolyLog}[3, -i e^{i (e+fx)}]}{f^3} + \frac{12 a^2 d^2 (c + dx) \operatorname{PolyLog}[3, i e^{i (e+fx)}]}{f^3} + \\ & \frac{3 a^2 d^3 \operatorname{PolyLog}[3, -e^{2 i (e+fx)}]}{2 f^4} - \frac{12 i a^2 d^3 \operatorname{PolyLog}[4, -i e^{i (e+fx)}]}{f^4} + \\ & \frac{12 i a^2 d^3 \operatorname{PolyLog}[4, i e^{i (e+fx)}]}{f^4} + \frac{a^2 (c + dx)^3 \operatorname{Tan}[e + fx]}{f} \end{aligned}$$

Result (type 4, 1027 leaves):

$$\begin{aligned}
& \frac{1}{16} x \left(4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) \cos[e+f x]^2 \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \sec[e+f x])^2 + \\
& \left(\cos[e+f x]^2 \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \sec[e+f x])^2 \right. \\
& \quad \left(c^3 \sin\left[\frac{f x}{2}\right] + 3 c^2 d x \sin\left[\frac{f x}{2}\right] + 3 c d^2 x^2 \sin\left[\frac{f x}{2}\right] + d^3 x^3 \sin\left[\frac{f x}{2}\right] \right) \Big) / \\
& \left(4 f \left(\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] - \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right) \right) + \\
& \left(\cos[e+f x]^2 \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \sec[e+f x])^2 \right. \\
& \quad \left(c^3 \sin\left[\frac{f x}{2}\right] + 3 c^2 d x \sin\left[\frac{f x}{2}\right] + 3 c d^2 x^2 \sin\left[\frac{f x}{2}\right] + d^3 x^3 \sin\left[\frac{f x}{2}\right] \right) \Big) / \\
& \left(4 f \left(\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right) \right) - \\
& \frac{1}{8 f^4} \operatorname{Int} \cos[e+f x]^2 \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^4 (a + a \sec[e+f x])^2 \\
& \left(6 c^2 d f^3 x + 6 c d^2 f^3 x^2 + 2 d^3 f^3 x^3 + 8 c^3 f^3 \operatorname{ArcTan}[\cos[e+f x] + i \sin[e+f x]] + \right. \\
& \quad 24 c^2 d f^3 x \operatorname{ArcTan}[\cos[e+f x] + i \sin[e+f x]] + 24 c d^2 f^3 x^2 \\
& \quad \operatorname{ArcTan}[\cos[e+f x] + i \sin[e+f x]] + 8 d^3 f^3 x^3 \operatorname{ArcTan}[\cos[e+f x] + i \sin[e+f x]] + \\
& \quad 6 i c^2 d f^2 \log[1 + \cos[2(e+f x)] + i \sin[2(e+f x)]] + \\
& \quad 12 i c d^2 f^2 x \log[1 + \cos[2(e+f x)] + i \sin[2(e+f x)]] + \\
& \quad 6 i d^3 f^2 x^2 \log[1 + \cos[2(e+f x)] + i \sin[2(e+f x)]] + \\
& \quad 12 d f^2 (c + d x)^2 \operatorname{PolyLog}[2, i \cos[e+f x] - \sin[e+f x]] - \\
& \quad 12 d f^2 (c + d x)^2 \operatorname{PolyLog}[2, -i \cos[e+f x] + \sin[e+f x]] + \\
& \quad 6 c d^2 f \operatorname{PolyLog}[2, -\cos[2(e+f x)] - i \sin[2(e+f x)]] + \\
& \quad 6 d^3 f x \operatorname{PolyLog}[2, -\cos[2(e+f x)] - i \sin[2(e+f x)]] + \\
& \quad 24 i c d^2 f \operatorname{PolyLog}[3, i \cos[e+f x] - \sin[e+f x]] + \\
& \quad 24 i d^3 f x \operatorname{PolyLog}[3, i \cos[e+f x] - \sin[e+f x]] - \\
& \quad 24 i c d^2 f \operatorname{PolyLog}[3, -i \cos[e+f x] + \sin[e+f x]] - \\
& \quad 24 i d^3 f x \operatorname{PolyLog}[3, -i \cos[e+f x] + \sin[e+f x]] + \\
& \quad 3 i d^3 \operatorname{PolyLog}[3, -\cos[2(e+f x)] - i \sin[2(e+f x)]] - 24 d^3 \\
& \quad \operatorname{PolyLog}[4, i \cos[e+f x] - \sin[e+f x]] + 24 d^3 \operatorname{PolyLog}[4, -i \cos[e+f x] + \sin[e+f x]] + \\
& \quad \left. 6 i c^2 d f^3 x \tan[e] + 6 i c d^2 f^3 x^2 \tan[e] + 2 i d^3 f^3 x^3 \tan[e] \right)
\end{aligned}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 (a + a \sec[e + f x])^2 dx$$

Optimal (type 4, 262 leaves, 14 steps):

$$\begin{aligned}
& -\frac{\frac{i}{2} a^2 (c+d x)^2}{f} + \frac{a^2 (c+d x)^3}{3 d} - \frac{4 \frac{i}{2} a^2 (c+d x)^2 \operatorname{ArcTan}[e^{i(e+f x)}]}{f} + \\
& \frac{2 a^2 d (c+d x) \log[1+e^{2 i(e+f x)}]}{f^2} + \frac{4 \frac{i}{2} a^2 d (c+d x) \operatorname{PolyLog}[2, -i e^{i(e+f x)}]}{f^2} - \\
& \frac{4 \frac{i}{2} a^2 d (c+d x) \operatorname{PolyLog}[2, i e^{i(e+f x)}]}{f^2} - \frac{i a^2 d^2 \operatorname{PolyLog}[2, -e^{2 i(e+f x)}]}{f^3} - \\
& \frac{4 a^2 d^2 \operatorname{PolyLog}[3, -i e^{i(e+f x)}]}{f^3} + \frac{4 a^2 d^2 \operatorname{PolyLog}[3, i e^{i(e+f x)}]}{f^3} + \frac{a^2 (c+d x)^2 \tan[e+f x]}{f}
\end{aligned}$$

Result (type 4, 685 leaves):

$$\begin{aligned}
& \frac{1}{12} x (3 c^2 + 3 c d x + d^2 x^2) \cos[e+f x]^2 \sec[\frac{e}{2} + \frac{f x}{2}]^4 (a + a \sec[e+f x])^2 + \\
& \cos[e+f x]^2 \sec[\frac{e}{2} + \frac{f x}{2}]^4 (a + a \sec[e+f x])^2 (c^2 \sin[\frac{f x}{2}] + 2 c d x \sin[\frac{f x}{2}] + d^2 x^2 \sin[\frac{f x}{2}]) + \\
& 4 f (\cos[\frac{e}{2}] - \sin[\frac{e}{2}]) (\cos[\frac{e}{2} + \frac{f x}{2}] - \sin[\frac{e}{2} + \frac{f x}{2}]) \\
& \cos[e+f x]^2 \sec[\frac{e}{2} + \frac{f x}{2}]^4 (a + a \sec[e+f x])^2 (c^2 \sin[\frac{f x}{2}] + 2 c d x \sin[\frac{f x}{2}] + d^2 x^2 \sin[\frac{f x}{2}]) - \\
& 4 f (\cos[\frac{e}{2}] + \sin[\frac{e}{2}]) (\cos[\frac{e}{2} + \frac{f x}{2}] + \sin[\frac{e}{2} + \frac{f x}{2}]) \\
& \frac{1}{4 f^3} i \cos[e+f x]^2 \sec[\frac{e}{2} + \frac{f x}{2}]^4 (a + a \sec[e+f x])^2 \\
& (2 c d f^2 x + d^2 f^2 x^2 + 4 c^2 f^2 \operatorname{ArcTan}[\cos[e+f x] + i \sin[e+f x]] + \\
& 8 c d f^2 x \operatorname{ArcTan}[\cos[e+f x] + i \sin[e+f x]] + 4 d^2 f^2 x^2 \\
& \operatorname{ArcTan}[\cos[e+f x] + i \sin[e+f x]] + 2 i c d f \log[1 + \cos[2(e+f x)] + i \sin[2(e+f x)]] + \\
& 2 i d^2 f x \log[1 + \cos[2(e+f x)] + i \sin[2(e+f x)]] + \\
& 4 d f (c+d x) \operatorname{PolyLog}[2, i \cos[e+f x] - \sin[e+f x]] - \\
& 4 d f (c+d x) \operatorname{PolyLog}[2, -i \cos[e+f x] + \sin[e+f x]] + d^2 \operatorname{PolyLog}[2, \\
& -\cos[2(e+f x)] - i \sin[2(e+f x)]] + 4 i d^2 \operatorname{PolyLog}[3, i \cos[e+f x] - \sin[e+f x]] - \\
& 4 i d^2 \operatorname{PolyLog}[3, -i \cos[e+f x] + \sin[e+f x]] + 2 i c d f^2 x \tan[e] + i d^2 f^2 x^2 \tan[e])
\end{aligned}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int (c+d x) (a + a \sec[e+f x])^2 dx$$

Optimal (type 4, 134 leaves, 9 steps):

$$\begin{aligned}
& \frac{a^2 (c+d x)^2}{2 d} - \frac{4 \frac{i}{2} a^2 (c+d x) \operatorname{ArcTan}[e^{i(e+f x)}]}{f} + \frac{a^2 d \log[\cos[e+f x]]}{f^2} + \\
& \frac{2 \frac{i}{2} a^2 d \operatorname{PolyLog}[2, -i e^{i(e+f x)}]}{f^2} - \frac{2 \frac{i}{2} a^2 d \operatorname{PolyLog}[2, i e^{i(e+f x)}]}{f^2} + \frac{a^2 (c+d x) \tan[e+f x]}{f}
\end{aligned}$$

Result (type 4, 728 leaves):

$$\begin{aligned}
& \left(x \cos[e + f x]^2 \sec[\frac{e}{2} + \frac{f x}{2}]^4 (a + a \sec[e + f x])^2 (2 c f \cos[e] + d f x \cos[e] + 2 d \sin[e]) \right) / \\
& \quad \left(8 f \left(\cos[\frac{e}{2}] - \sin[\frac{e}{2}] \right) \left(\cos[\frac{e}{2}] + \sin[\frac{e}{2}] \right) \right) + \\
& \quad \left(d \cos[e + f x]^2 \sec[e] \sec[\frac{e}{2} + \frac{f x}{2}]^4 (a + a \sec[e + f x])^2 \right. \\
& \quad \left. (\cos[e] \log[\cos[e] \cos[f x] - \sin[e] \sin[f x]] + f x \sin[e]) \right) / (4 f^2 (\cos[e]^2 + \sin[e]^2)) + \\
& \left(i c \operatorname{ArcTan}\left[\frac{-i \sin[e] - i \cos[e] \tan[\frac{f x}{2}]}{\sqrt{\cos[e]^2 + \sin[e]^2}} \right] \cos[e + f x]^2 \sec[\frac{e}{2} + \frac{f x}{2}]^4 (a + a \sec[e + f x])^2 \right) / \\
& \quad \left(f \sqrt{\cos[e]^2 + \sin[e]^2} \right) + \frac{1}{2 f^2} d \cos[e + f x]^2 \sec[\frac{e}{2} + \frac{f x}{2}]^4 (a + a \sec[e + f x])^2 \left(-\frac{1}{\sqrt{1 + \cot[e]^2}} \right. \\
& \quad \left. \csc[e] ((f x - \operatorname{ArcTan}[\cot[e]]) (\log[1 - e^{i(f x - \operatorname{ArcTan}[\cot[e])}] - \log[1 + e^{i(f x - \operatorname{ArcTan}[\cot[e])}]] + \right. \\
& \quad \left. i (\operatorname{PolyLog}[2, -e^{i(f x - \operatorname{ArcTan}[\cot[e])}] - \operatorname{PolyLog}[2, e^{i(f x - \operatorname{ArcTan}[\cot[e])}]))) + \right. \\
& \quad \left. \frac{2 \operatorname{ArcTan}[\cot[e]] \operatorname{ArcTanh}\left[\frac{\sin[e] + \cos[e] \tan[\frac{f x}{2}]}{\sqrt{\cos[e]^2 + \sin[e]^2}} \right]}{\sqrt{\cos[e]^2 + \sin[e]^2}} \right) + \\
& \quad \left(\cos[e + f x]^2 \sec[\frac{e}{2} + \frac{f x}{2}]^4 (a + a \sec[e + f x])^2 \left(c \sin[\frac{f x}{2}] + d x \sin[\frac{f x}{2}] \right) \right) / \\
& \quad \left(4 f \left(\cos[\frac{e}{2}] - \sin[\frac{e}{2}] \right) \left(\cos[\frac{e}{2} + \frac{f x}{2}] - \sin[\frac{e}{2} + \frac{f x}{2}] \right) \right) + \\
& \quad \left(\cos[e + f x]^2 \sec[\frac{e}{2} + \frac{f x}{2}]^4 (a + a \sec[e + f x])^2 \left(c \sin[\frac{f x}{2}] + d x \sin[\frac{f x}{2}] \right) \right) / \\
& \quad \left(4 f \left(\cos[\frac{e}{2}] + \sin[\frac{e}{2}] \right) \left(\cos[\frac{e}{2} + \frac{f x}{2}] + \sin[\frac{e}{2} + \frac{f x}{2}] \right) \right) - \\
& \quad \frac{d x \cos[e + f x]^2 \sec[\frac{e}{2} + \frac{f x}{2}]^4 (a + a \sec[e + f x])^2 \tan[e]}{4 f}
\end{aligned}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^2}{a + a \sec[e + f x]} dx$$

Optimal (type 4, 119 leaves, 8 steps):

$$\begin{aligned}
& \frac{i (c + d x)^2}{a f} + \frac{(c + d x)^3}{3 a d} - \frac{4 d (c + d x) \log[1 + e^{i(e+f x)}]}{a f^2} + \\
& \frac{4 i d^2 \operatorname{PolyLog}[2, -e^{i(e+f x)}]}{a f^3} - \frac{(c + d x)^2 \tan[\frac{e}{2} + \frac{f x}{2}]}{a f}
\end{aligned}$$

Result (type 4, 528 leaves):

$$\begin{aligned}
 & \frac{2 \times (3 c^2 + 3 c d x + d^2 x^2) \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \sec[e + f x]}{3 (a + a \sec[e + f x])} - \left(8 c d \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \sec\left[\frac{e}{2}\right]\right. \\
 & \quad \left.\sec[e + f x] \left(\cos\left[\frac{e}{2}\right] \log\left[\cos\left[\frac{e}{2}\right] \cos\left[\frac{f x}{2}\right] - \sin\left[\frac{e}{2}\right] \sin\left[\frac{f x}{2}\right]\right] + \frac{1}{2} f x \sin\left[\frac{e}{2}\right]\right)\right)/ \\
 & \quad \left(f^2 (a + a \sec[e + f x]) \left(\cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2\right)\right) - \\
 & \quad \left(8 d^2 \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \csc\left[\frac{e}{2}\right] \left(\frac{1}{4} e^{-i \operatorname{ArcTan}[\cot\left[\frac{e}{2}\right]]} f^2 x^2 - \frac{1}{\sqrt{1 + \cot\left[\frac{e}{2}\right]^2}}\right.\right. \\
 & \quad \left.\cot\left[\frac{e}{2}\right] \left(\frac{1}{2} i f x \left(-\pi - 2 \operatorname{ArcTan}[\cot\left[\frac{e}{2}\right]]\right) - \pi \log[1 + e^{-i f x}] - 2 \left(\frac{f x}{2} - \operatorname{ArcTan}[\cot\left[\frac{e}{2}\right]]\right)\right.\right. \\
 & \quad \left.\log[1 - e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}[\cot\left[\frac{e}{2}\right]]\right)}] + \pi \log[\cos\left[\frac{f x}{2}\right]] - 2 \operatorname{ArcTan}[\cot\left[\frac{e}{2}\right]]\right.\right. \\
 & \quad \left.\log\left[\sin\left[\frac{f x}{2} - \operatorname{ArcTan}[\cot\left[\frac{e}{2}\right]]\right]\right] + i \operatorname{PolyLog}[2, e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}[\cot\left[\frac{e}{2}\right]]\right)}]\right)\right) \\
 & \quad \left.\sec\left[\frac{e}{2}\right] \sec[e + f x]\right)/ \left(f^3 (a + a \sec[e + f x]) \sqrt{\csc\left[\frac{e}{2}\right]^2 \left(\cos\left[\frac{e}{2}\right]^2 + \sin\left[\frac{e}{2}\right]^2\right)}\right) - \\
 & \quad \left(2 \cos\left[\frac{e}{2} + \frac{f x}{2}\right] \sec\left[\frac{e}{2}\right] \sec[e + f x] \left(c^2 \sin\left[\frac{f x}{2}\right] + 2 c d x \sin\left[\frac{f x}{2}\right] + d^2 x^2 \sin\left[\frac{f x}{2}\right]\right)\right)/ \\
 & \quad \left(f (a + a \sec[e + f x])\right)
 \end{aligned}$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+dx)^3}{(a+a \sec[e+f x])^2} dx$$

Optimal (type 4, 288 leaves, 19 steps):

$$\begin{aligned}
 & \frac{5 i (c + d x)^3}{3 a^2 f} + \frac{(c + d x)^4}{4 a^2 d} - \frac{10 d (c + d x)^2 \log[1 + e^{i (e + f x)}]}{a^2 f^2} + \frac{4 d^3 \log[\cos\left[\frac{e}{2} + \frac{f x}{2}\right]]}{a^2 f^4} + \\
 & \frac{20 i d^2 (c + d x) \operatorname{PolyLog}[2, -e^{i (e + f x)}]}{a^2 f^3} - \frac{20 d^3 \operatorname{PolyLog}[3, -e^{i (e + f x)}]}{a^2 f^4} - \frac{d (c + d x)^2 \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^2}{2 a^2 f^2} + \\
 & \frac{2 d^2 (c + d x) \tan\left[\frac{e}{2} + \frac{f x}{2}\right]}{a^2 f^3} - \frac{5 (c + d x)^3 \tan\left[\frac{e}{2} + \frac{f x}{2}\right]}{3 a^2 f} + \frac{(c + d x)^3 \sec\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \tan\left[\frac{e}{2} + \frac{f x}{2}\right]}{6 a^2 f}
 \end{aligned}$$

Result (type 4, 1455 leaves) :

$$\begin{aligned}
& \left(20 d^3 e^{-\frac{i e}{2}} \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \right. \\
& \quad \left(\frac{i}{2} f^2 x^2 (e^{i e} f x + 3 i (1 + e^{i e}) \log [1 + e^{i (e+f x)}]) + 6 i (1 + e^{i e}) f x \text{PolyLog}[2, -e^{i (e+f x)}] - \right. \\
& \quad \left. 6 (1 + e^{i e}) \text{PolyLog}[3, -e^{i (e+f x)}] \right) \sec \left[\frac{e}{2} \right] \sec [e + f x]^2 \Big) / \\
& \quad \left(3 f^4 (a + a \sec [e + f x])^2 \right) + \left(16 d^3 \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \sec \left[\frac{e}{2} \right] \sec [e + f x]^2 \right. \\
& \quad \left(\cos \left[\frac{e}{2} \right] \log [\cos \left[\frac{e}{2} \right] \cos \left[\frac{f x}{2} \right] - \sin \left[\frac{e}{2} \right] \sin \left[\frac{f x}{2} \right]] + \frac{1}{2} f x \sin \left[\frac{e}{2} \right] \right) \Big) / \\
& \quad \left(f^4 (a + a \sec [e + f x])^2 \left(\cos \left[\frac{e}{2} \right]^2 + \sin \left[\frac{e}{2} \right]^2 \right) - \right. \\
& \quad \left(40 c^2 d \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \sec \left[\frac{e}{2} \right] \sec [e + f x]^2 \right. \\
& \quad \left(\cos \left[\frac{e}{2} \right] \log [\cos \left[\frac{e}{2} \right] \cos \left[\frac{f x}{2} \right] - \sin \left[\frac{e}{2} \right] \sin \left[\frac{f x}{2} \right]] + \frac{1}{2} f x \sin \left[\frac{e}{2} \right] \right) \Big) / \\
& \quad \left(f^2 (a + a \sec [e + f x])^2 \left(\cos \left[\frac{e}{2} \right]^2 + \sin \left[\frac{e}{2} \right]^2 \right) - \right. \\
& \quad \left(80 c d^2 \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \csc \left[\frac{e}{2} \right] \left(\frac{1}{4} e^{-i \text{ArcTan}[\cot[\frac{e}{2}]]} f^2 x^2 - \frac{1}{\sqrt{1 + \cot[\frac{e}{2}]^2}} \right. \right. \\
& \quad \left. \left. \cot \left[\frac{e}{2} \right] \left(\frac{1}{2} i f x \left(-\pi - 2 \text{ArcTan}[\cot[\frac{e}{2}]] \right) - \pi \log [1 + e^{-i f x}] - 2 \left(\frac{f x}{2} - \text{ArcTan}[\cot[\frac{e}{2}]] \right) \right. \right. \\
& \quad \left. \left. \log [1 - e^{2 i \left(\frac{f x}{2} - \text{ArcTan}[\cot[\frac{e}{2}]] \right)}] + \pi \log [\cos \left[\frac{f x}{2} \right]] - 2 \text{ArcTan}[\cot[\frac{e}{2}]] \right. \right. \\
& \quad \left. \left. \log \left[\sin \left[\frac{f x}{2} - \text{ArcTan}[\cot[\frac{e}{2}]] \right] \right] + i \text{PolyLog}[2, e^{2 i \left(\frac{f x}{2} - \text{ArcTan}[\cot[\frac{e}{2}]] \right)}] \right) \right) \Big) \\
& \quad \left. \left(\sec \left[\frac{e}{2} \right] \sec [e + f x]^2 \right) \Big) / \left(f^3 (a + a \sec [e + f x])^2 \sqrt{\csc \left[\frac{e}{2} \right]^2 \left(\cos \left[\frac{e}{2} \right]^2 + \sin \left[\frac{e}{2} \right]^2 \right)} + \right. \\
& \quad \left. \frac{1}{24 f^3 (a + a \sec [e + f x])^2} \cos \left[\frac{e}{2} + \frac{f x}{2} \right] \sec \left[\frac{e}{2} \right] \sec [e + f x]^2 \right. \\
& \quad \left(-24 c^2 d f \cos \left[\frac{f x}{2} \right] - 48 c d^2 f x \cos \left[\frac{f x}{2} \right] + 36 c^3 f^3 x \cos \left[\frac{f x}{2} \right] - 24 d^3 f x^2 \cos \left[\frac{f x}{2} \right] + \right. \\
& \quad \left. 54 c^2 d f^3 x^2 \cos \left[\frac{f x}{2} \right] + 36 c d^2 f^3 x^3 \cos \left[\frac{f x}{2} \right] + 9 d^3 f^3 x^4 \cos \left[\frac{f x}{2} \right] - 24 c^2 d f \cos \left[e + \frac{f x}{2} \right] - \right. \\
& \quad \left. 48 c d^2 f x \cos \left[e + \frac{f x}{2} \right] + 36 c^3 f^3 x \cos \left[e + \frac{f x}{2} \right] - 24 d^3 f x^2 \cos \left[e + \frac{f x}{2} \right] + \right)
\end{aligned}$$

$$\begin{aligned}
& 54 c^2 d f^3 x^2 \cos \left[e + \frac{f x}{2} \right] + 36 c d^2 f^3 x^3 \cos \left[e + \frac{f x}{2} \right] + 9 d^3 f^3 x^4 \cos \left[e + \frac{f x}{2} \right] + \\
& 12 c^3 f^3 x \cos \left[e + \frac{3 f x}{2} \right] + 18 c^2 d f^3 x^2 \cos \left[e + \frac{3 f x}{2} \right] + 12 c d^2 f^3 x^3 \cos \left[e + \frac{3 f x}{2} \right] + \\
& 3 d^3 f^3 x^4 \cos \left[e + \frac{3 f x}{2} \right] + 12 c^3 f^3 x \cos \left[2 e + \frac{3 f x}{2} \right] + 18 c^2 d f^3 x^2 \cos \left[2 e + \frac{3 f x}{2} \right] + \\
& 12 c d^2 f^3 x^3 \cos \left[2 e + \frac{3 f x}{2} \right] + 3 d^3 f^3 x^4 \cos \left[2 e + \frac{3 f x}{2} \right] + 96 c d^2 \sin \left[\frac{f x}{2} \right] - \\
& 72 c^3 f^2 \sin \left[\frac{f x}{2} \right] + 96 d^3 x \sin \left[\frac{f x}{2} \right] - 216 c^2 d f^2 x \sin \left[\frac{f x}{2} \right] - 216 c d^2 f^2 x^2 \sin \left[\frac{f x}{2} \right] - \\
& 72 d^3 f^2 x^3 \sin \left[\frac{f x}{2} \right] - 48 c d^2 \sin \left[e + \frac{f x}{2} \right] + 48 c^3 f^2 \sin \left[e + \frac{f x}{2} \right] - 48 d^3 x \sin \left[e + \frac{f x}{2} \right] + \\
& 144 c^2 d f^2 x \sin \left[e + \frac{f x}{2} \right] + 144 c d^2 f^2 x^2 \sin \left[e + \frac{f x}{2} \right] + 48 d^3 f^2 x^3 \sin \left[e + \frac{f x}{2} \right] + \\
& 48 c d^2 \sin \left[e + \frac{3 f x}{2} \right] - 40 c^3 f^2 \sin \left[e + \frac{3 f x}{2} \right] + 48 d^3 x \sin \left[e + \frac{3 f x}{2} \right] - \\
& 120 c^2 d f^2 x \sin \left[e + \frac{3 f x}{2} \right] - 120 c d^2 f^2 x^2 \sin \left[e + \frac{3 f x}{2} \right] - 40 d^3 f^2 x^3 \sin \left[e + \frac{3 f x}{2} \right]
\end{aligned}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+d x)^2}{(a+a \sec[e+f x])^2} dx$$

Optimal (type 4, 229 leaves, 17 steps):

$$\begin{aligned}
& \frac{5 i (c+d x)^2}{3 a^2 f} + \frac{(c+d x)^3}{3 a^2 d} - \frac{20 d (c+d x) \log[1+e^{i(e+f x)}]}{3 a^2 f^2} + \\
& \frac{20 i d^2 \text{PolyLog}[2, -e^{i(e+f x)}]}{3 a^2 f^3} - \frac{d (c+d x) \sec[\frac{e}{2} + \frac{f x}{2}]^2}{3 a^2 f^2} + \frac{2 d^2 \tan[\frac{e}{2} + \frac{f x}{2}]}{3 a^2 f^3} - \\
& \frac{5 (c+d x)^2 \tan[\frac{e}{2} + \frac{f x}{2}]}{3 a^2 f} + \frac{(c+d x)^2 \sec[\frac{e}{2} + \frac{f x}{2}]^2 \tan[\frac{e}{2} + \frac{f x}{2}]}{6 a^2 f}
\end{aligned}$$

Result (type 4, 925 leaves):

$$\begin{aligned}
& - \left(\left(80 c d \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \sec \left[\frac{e}{2} \right] \sec [e + f x]^2 \right. \right. \\
& \quad \left(\cos \left[\frac{e}{2} \right] \log [\cos \left[\frac{e}{2} \right] \cos \left[\frac{f x}{2} \right] - \sin \left[\frac{e}{2} \right] \sin \left[\frac{f x}{2} \right]] + \frac{1}{2} f x \sin \left[\frac{e}{2} \right] \right) \Big) \Big) \Big/ \\
& \quad \left(3 f^2 (a + a \sec [e + f x])^2 \left(\cos \left[\frac{e}{2} \right]^2 + \sin \left[\frac{e}{2} \right]^2 \right) \right) \Big) - \\
& \quad \left(80 d^2 \cos \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \csc \left[\frac{e}{2} \right] \left(\frac{1}{4} e^{-i \operatorname{ArcTan}[\cot \left[\frac{e}{2} \right]]} f^2 x^2 - \frac{1}{\sqrt{1 + \cot \left[\frac{e}{2} \right]^2}} \right. \right. \\
& \quad \left. \left. \cot \left[\frac{e}{2} \right] \left(\frac{1}{2} i f x \left(-\pi - 2 \operatorname{ArcTan}[\cot \left[\frac{e}{2} \right]] \right) - \pi \log [1 + e^{-i f x}] - 2 \left(\frac{f x}{2} - \operatorname{ArcTan}[\cot \left[\frac{e}{2} \right]] \right) \right. \right. \\
& \quad \left. \left. \log [1 - e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}[\cot \left[\frac{e}{2} \right]] \right)}] + \pi \log [\cos \left[\frac{f x}{2} \right]] - 2 \operatorname{ArcTan}[\cot \left[\frac{e}{2} \right]] \right. \right. \\
& \quad \left. \left. \log [\sin \left[\frac{f x}{2} - \operatorname{ArcTan}[\cot \left[\frac{e}{2} \right]] \right]] + i \operatorname{PolyLog}[2, e^{2 i \left(\frac{f x}{2} - \operatorname{ArcTan}[\cot \left[\frac{e}{2} \right]] \right)}] \right) \right) \sec \left[\frac{e}{2} \right] \right. \\
& \quad \left. \sec [e + f x]^2 \right) \Big/ \left(3 f^3 (a + a \sec [e + f x])^2 \sqrt{\csc \left[\frac{e}{2} \right]^2 \left(\cos \left[\frac{e}{2} \right]^2 + \sin \left[\frac{e}{2} \right]^2 \right)} \right) + \\
& \quad \frac{1}{6 f^3 (a + a \sec [e + f x])^2} \cos \left[\frac{e}{2} + \frac{f x}{2} \right] \sec \left[\frac{e}{2} \right] \sec [e + f x]^2 \\
& \quad \left(-4 c d f \cos \left[\frac{f x}{2} \right] - 4 d^2 f x \cos \left[\frac{f x}{2} \right] + 9 c^2 f^3 x \cos \left[\frac{f x}{2} \right] + 9 c d f^3 x^2 \cos \left[\frac{f x}{2} \right] + \right. \\
& \quad 3 d^2 f^3 x^3 \cos \left[\frac{f x}{2} \right] - 4 c d f \cos \left[e + \frac{f x}{2} \right] - 4 d^2 f x \cos \left[e + \frac{f x}{2} \right] + 9 c^2 f^3 x \cos \left[e + \frac{f x}{2} \right] + \\
& \quad 9 c d f^3 x^2 \cos \left[e + \frac{f x}{2} \right] + 3 d^2 f^3 x^3 \cos \left[e + \frac{f x}{2} \right] + 3 c^2 f^3 x \cos \left[e + \frac{3 f x}{2} \right] + \\
& \quad 3 c d f^3 x^2 \cos \left[e + \frac{3 f x}{2} \right] + d^2 f^3 x^3 \cos \left[e + \frac{3 f x}{2} \right] + 3 c^2 f^3 x \cos \left[2 e + \frac{3 f x}{2} \right] + \\
& \quad 3 c d f^3 x^2 \cos \left[2 e + \frac{3 f x}{2} \right] + d^2 f^3 x^3 \cos \left[2 e + \frac{3 f x}{2} \right] + 8 d^2 \sin \left[\frac{f x}{2} \right] - 18 c^2 f^2 \sin \left[\frac{f x}{2} \right] - \\
& \quad 36 c d f^2 x \sin \left[\frac{f x}{2} \right] - 18 d^2 f^2 x^2 \sin \left[\frac{f x}{2} \right] - 4 d^2 \sin \left[e + \frac{f x}{2} \right] + 12 c^2 f^2 \sin \left[e + \frac{f x}{2} \right] + \\
& \quad 24 c d f^2 x \sin \left[e + \frac{f x}{2} \right] + 12 d^2 f^2 x^2 \sin \left[e + \frac{f x}{2} \right] + 4 d^2 \sin \left[e + \frac{3 f x}{2} \right] - \\
& \quad \left. 10 c^2 f^2 \sin \left[e + \frac{3 f x}{2} \right] - 20 c d f^2 x \sin \left[e + \frac{3 f x}{2} \right] - 10 d^2 f^2 x^2 \sin \left[e + \frac{3 f x}{2} \right] \right)
\end{aligned}$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 (a + b \sec[e + f x]) dx$$

Optimal (type 4, 227 leaves, 11 steps):

$$\begin{aligned} & \frac{a (c + d x)^4}{4 d} - \frac{2 i b (c + d x)^3 \operatorname{ArcTan}[e^{i (e+f x)}]}{f} + \\ & \frac{3 i b d (c + d x)^2 \operatorname{PolyLog}[2, -i e^{i (e+f x)}]}{f^2} - \frac{3 i b d (c + d x)^2 \operatorname{PolyLog}[2, i e^{i (e+f x)}]}{f^2} - \\ & \frac{6 b d^2 (c + d x) \operatorname{PolyLog}[3, -i e^{i (e+f x)}]}{f^3} + \frac{6 b d^2 (c + d x) \operatorname{PolyLog}[3, i e^{i (e+f x)}]}{f^3} - \\ & \frac{6 i b d^3 \operatorname{PolyLog}[4, -i e^{i (e+f x)}]}{f^4} + \frac{6 i b d^3 \operatorname{PolyLog}[4, i e^{i (e+f x)}]}{f^4} \end{aligned}$$

Result (type 4, 474 leaves):

$$\begin{aligned} & \frac{1}{4 f^4} \left(4 a c^3 f^4 x + 6 a c^2 d f^4 x^2 + 4 a c d^2 f^4 x^3 + a d^3 f^4 x^4 - 8 i b c^3 f^3 \operatorname{ArcTan}[e^{i (e+f x)}] + \right. \\ & 12 b c^2 d f^3 x \operatorname{Log}[1 - i e^{i (e+f x)}] + 12 b c d^2 f^3 x^2 \operatorname{Log}[1 - i e^{i (e+f x)}] + \\ & 4 b d^3 f^3 x^3 \operatorname{Log}[1 - i e^{i (e+f x)}] - 12 b c^2 d f^3 x \operatorname{Log}[1 + i e^{i (e+f x)}] - \\ & 12 b c d^2 f^3 x^2 \operatorname{Log}[1 + i e^{i (e+f x)}] - 4 b d^3 f^3 x^3 \operatorname{Log}[1 + i e^{i (e+f x)}] + \\ & 12 i b d f^2 (c + d x)^2 \operatorname{PolyLog}[2, -i e^{i (e+f x)}] - 12 i b d f^2 (c + d x)^2 \operatorname{PolyLog}[2, i e^{i (e+f x)}] - \\ & 24 b c d^2 f \operatorname{PolyLog}[3, -i e^{i (e+f x)}] - 24 b d^3 f x \operatorname{PolyLog}[3, -i e^{i (e+f x)}] + \\ & 24 b c d^2 f \operatorname{PolyLog}[3, i e^{i (e+f x)}] + 24 b d^3 f x \operatorname{PolyLog}[3, i e^{i (e+f x)}] - \\ & \left. 24 i b d^3 \operatorname{PolyLog}[4, -i e^{i (e+f x)}] + 24 i b d^3 \operatorname{PolyLog}[4, i e^{i (e+f x)}] \right) \end{aligned}$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int (c + d x) (a + b \sec[e + f x]) dx$$

Optimal (type 4, 93 leaves, 7 steps):

$$\begin{aligned} & \frac{a (c + d x)^2}{2 d} - \frac{2 i b (c + d x) \operatorname{ArcTan}[e^{i (e+f x)}]}{f} + \\ & \frac{i b d \operatorname{PolyLog}[2, -i e^{i (e+f x)}]}{f^2} - \frac{i b d \operatorname{PolyLog}[2, i e^{i (e+f x)}]}{f^2} \end{aligned}$$

Result (type 4, 236 leaves):

$$\begin{aligned} & a c x + \frac{1}{2} a d x^2 - \frac{b c \operatorname{Log}[\cos[\frac{e}{2} + \frac{f x}{2}] - \sin[\frac{e}{2} + \frac{f x}{2}]]}{f} + \frac{b c \operatorname{Log}[\cos[\frac{e}{2} + \frac{f x}{2}] + \sin[\frac{e}{2} + \frac{f x}{2}]]}{f} + \\ & \frac{1}{f^2} b d \left(\left(-e + \frac{\pi}{2} - f x \right) \left(\operatorname{Log}[1 - e^{i (-e + \frac{\pi}{2} - f x)}] - \operatorname{Log}[1 + e^{i (-e + \frac{\pi}{2} - f x)}] \right) - \left(-e + \frac{\pi}{2} \right) \right. \\ & \left. \operatorname{Log}[\tan[\frac{1}{2} \left(-e + \frac{\pi}{2} - f x \right)]] + i \left(\operatorname{PolyLog}[2, -e^{i (-e + \frac{\pi}{2} - f x)}] - \operatorname{PolyLog}[2, e^{i (-e + \frac{\pi}{2} - f x)}] \right) \right) \end{aligned}$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 (a + b \sec[e + f x])^2 dx$$

Optimal (type 4, 364 leaves, 17 steps):

$$\begin{aligned}
& -\frac{\frac{i b^2 (c + d x)^3}{f} + \frac{a^2 (c + d x)^4}{4 d} - \frac{4 i a b (c + d x)^3 \operatorname{ArcTan}[e^{i (e+f x)}]}{f} +}{f} \\
& + \frac{3 b^2 d (c + d x)^2 \log[1 + e^{2 i (e+f x)}]}{f^2} + \frac{6 i a b d (c + d x)^2 \operatorname{PolyLog}[2, -i e^{i (e+f x)}]}{f^2} - \\
& \frac{6 i a b d (c + d x)^2 \operatorname{PolyLog}[2, i e^{i (e+f x)}]}{f^2} - \frac{3 i b^2 d^2 (c + d x) \operatorname{PolyLog}[2, -e^{2 i (e+f x)}]}{f^3} - \\
& \frac{12 a b d^2 (c + d x) \operatorname{PolyLog}[3, -i e^{i (e+f x)}]}{f^3} + \frac{12 a b d^2 (c + d x) \operatorname{PolyLog}[3, i e^{i (e+f x)}]}{f^3} + \\
& \frac{3 b^2 d^3 \operatorname{PolyLog}[3, -e^{2 i (e+f x)}]}{2 f^4} - \frac{12 i a b d^3 \operatorname{PolyLog}[4, -i e^{i (e+f x)}]}{f^4} + \\
& \frac{12 i a b d^3 \operatorname{PolyLog}[4, i e^{i (e+f x)}]}{f^4} + \frac{b^2 (c + d x)^3 \tan[e + f x]}{f}
\end{aligned}$$

Result (type 4, 1700 leaves):

$$\begin{aligned}
& \frac{a^2 x \left(4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3\right) \cos[e+f x]^2 (a+b \sec[e+f x])^2}{4 (b+a \cos[e+f x])^2} + \\
& \frac{1}{2 (1+e^{2 i e}) f^4 (b+a \cos[e+f x])^2} b \cos[e+f x]^2 \\
& \left(-12 i b c^2 d e^{2 i e} f^3 x - 12 i b c d^2 e^{2 i e} f^3 x^2 - 4 i b d^3 e^{2 i e} f^3 x^3 - 8 i a c^3 f^3 \operatorname{ArcTan}[e^{i (e+f x)}] - \right. \\
& 8 i a c^3 e^{2 i e} f^3 \operatorname{ArcTan}[e^{i (e+f x)}] + 12 a c^2 d f^3 x \log[1-i e^{i (e+f x)}] + \\
& 12 a c^2 d e^{2 i e} f^3 x \log[1-i e^{i (e+f x)}] + 12 a c d^2 f^3 x^2 \log[1-i e^{i (e+f x)}] + \\
& 12 a c d^2 e^{2 i e} f^3 x^2 \log[1-i e^{i (e+f x)}] + 4 a d^3 f^3 x^3 \log[1-i e^{i (e+f x)}] + \\
& 4 a d^3 e^{2 i e} f^3 x^3 \log[1-i e^{i (e+f x)}] - 12 a c^2 d f^3 x \log[1+i e^{i (e+f x)}] - \\
& 12 a c^2 d e^{2 i e} f^3 x \log[1+i e^{i (e+f x)}] - 12 a c d^2 f^3 x^2 \log[1+i e^{i (e+f x)}] - \\
& 12 a c d^2 e^{2 i e} f^3 x^2 \log[1+i e^{i (e+f x)}] - 4 a d^3 f^3 x^3 \log[1+i e^{i (e+f x)}] - \\
& 4 a d^3 e^{2 i e} f^3 x^3 \log[1+i e^{i (e+f x)}] + 6 b c^2 d f^2 \log[1+e^{2 i (e+f x)}] + \\
& 6 b c^2 d e^{2 i e} f^2 \log[1+e^{2 i (e+f x)}] + 12 b c d^2 f^2 x \log[1+e^{2 i (e+f x)}] + \\
& 12 b c d^2 e^{2 i e} f^2 x \log[1+e^{2 i (e+f x)}] + 6 b d^3 f^2 x^2 \log[1+e^{2 i (e+f x)}] + \\
& 6 b d^3 e^{2 i e} f^2 x^2 \log[1+e^{2 i (e+f x)}] + 12 i a d (1+e^{2 i e}) f^2 (c+d x)^2 \operatorname{PolyLog}[2, -i e^{i (e+f x)}] - \\
& 12 i a d (1+e^{2 i e}) f^2 (c+d x)^2 \operatorname{PolyLog}[2, i e^{i (e+f x)}] - \\
& 6 i b c d^2 f \operatorname{PolyLog}[2, -e^{2 i (e+f x)}] - 6 i b c d^2 e^{2 i e} f \operatorname{PolyLog}[2, -e^{2 i (e+f x)}] - \\
& 6 i b d^3 f x \operatorname{PolyLog}[2, -e^{2 i (e+f x)}] - 6 i b d^3 e^{2 i e} f x \operatorname{PolyLog}[2, -e^{2 i (e+f x)}] - \\
& 24 a c d^2 f \operatorname{PolyLog}[3, -i e^{i (e+f x)}] - 24 a c d^2 e^{2 i e} f \operatorname{PolyLog}[3, -i e^{i (e+f x)}] - \\
& 24 a d^3 f x \operatorname{PolyLog}[3, -i e^{i (e+f x)}] - 24 a d^3 e^{2 i e} f x \operatorname{PolyLog}[3, -i e^{i (e+f x)}] + \\
& 24 a c d^2 f \operatorname{PolyLog}[3, i e^{i (e+f x)}] + 24 a c d^2 e^{2 i e} f \operatorname{PolyLog}[3, i e^{i (e+f x)}] + \\
& 24 a d^3 f x \operatorname{PolyLog}[3, i e^{i (e+f x)}] + 24 a d^3 e^{2 i e} f x \operatorname{PolyLog}[3, i e^{i (e+f x)}] + \\
& 3 b d^3 \operatorname{PolyLog}[3, -e^{2 i (e+f x)}] + 3 b d^3 e^{2 i e} \operatorname{PolyLog}[3, -e^{2 i (e+f x)}] - \\
& 24 i a d^3 \operatorname{PolyLog}[4, -i e^{i (e+f x)}] - 24 i a d^3 e^{2 i e} \operatorname{PolyLog}[4, -i e^{i (e+f x)}] + \\
& 24 i a d^3 \operatorname{PolyLog}[4, i e^{i (e+f x)}] + 24 i a d^3 e^{2 i e} \operatorname{PolyLog}[4, i e^{i (e+f x)}] \Big) \\
& (a+b \sec[e+f x])^2 + \left(\cos[e+f x]^2 (a+b \sec[e+f x])^2 \right. \\
& \left(b^2 c^3 \sin\left[\frac{f x}{2}\right] + 3 b^2 c^2 d x \sin\left[\frac{f x}{2}\right] + 3 b^2 c d^2 x^2 \sin\left[\frac{f x}{2}\right] + b^2 d^3 x^3 \sin\left[\frac{f x}{2}\right] \right) / \\
& \left(f (b+a \cos[e+f x])^2 \left(\cos\left[\frac{e}{2}\right] - \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] - \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right) + \right. \\
& \left(\cos[e+f x]^2 (a+b \sec[e+f x])^2 \right. \\
& \left(b^2 c^3 \sin\left[\frac{f x}{2}\right] + 3 b^2 c^2 d x \sin\left[\frac{f x}{2}\right] + 3 b^2 c d^2 x^2 \sin\left[\frac{f x}{2}\right] + b^2 d^3 x^3 \sin\left[\frac{f x}{2}\right] \right) / \\
& \left. \left(f (b+a \cos[e+f x])^2 \left(\cos\left[\frac{e}{2}\right] + \sin\left[\frac{e}{2}\right] \right) \left(\cos\left[\frac{e}{2} + \frac{f x}{2}\right] + \sin\left[\frac{e}{2} + \frac{f x}{2}\right] \right) \right)
\end{aligned}$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^2 (a+b \sec[e+f x])^2 dx$$

Optimal (type 4, 257 leaves, 14 steps):

$$\begin{aligned}
& -\frac{\frac{i b^2 (c+dx)^2}{f} + \frac{a^2 (c+dx)^3}{3d} - \frac{4 i a b (c+dx)^2 \operatorname{ArcTan}[e^{i (e+fx)}]}{f} + \\
& \frac{2 b^2 d (c+dx) \operatorname{Log}[1+e^{2 i (e+fx)}]}{f^2} + \frac{4 i a b d (c+dx) \operatorname{PolyLog}[2, -i e^{i (e+fx)}]}{f^2} - \\
& \frac{4 i a b d (c+dx) \operatorname{PolyLog}[2, i e^{i (e+fx)}]}{f^2} - \frac{i b^2 d^2 \operatorname{PolyLog}[2, -e^{2 i (e+fx)}]}{f^3} - \\
& \frac{4 a b d^2 \operatorname{PolyLog}[3, -i e^{i (e+fx)}]}{f^3} + \frac{4 a b d^2 \operatorname{PolyLog}[3, i e^{i (e+fx)}]}{f^3} + \frac{b^2 (c+dx)^2 \operatorname{Tan}[e+fx]}{f}
\end{aligned}$$

Result (type 4, 703 leaves):

$$\begin{aligned}
& \frac{a^2 x (3 c^2 + 3 c d x + d^2 x^2) \cos[e+fx]^2 (a+b \sec[e+fx])^2}{3 (b+a \cos[e+fx])^2} + \\
& \left(\cos[e+fx]^2 (a+b \sec[e+fx])^2 \left(b^2 c^2 \sin[\frac{fx}{2}] + 2 b^2 c d x \sin[\frac{fx}{2}] + b^2 d^2 x^2 \sin[\frac{fx}{2}] \right) \right) / \\
& \left(f (b+a \cos[e+fx])^2 \left(\cos[\frac{e}{2}] - \sin[\frac{e}{2}] \right) \left(\cos[\frac{e}{2} + \frac{fx}{2}] - \sin[\frac{e}{2} + \frac{fx}{2}] \right) \right) + \\
& \left(\cos[e+fx]^2 (a+b \sec[e+fx])^2 \left(b^2 c^2 \sin[\frac{fx}{2}] + 2 b^2 c d x \sin[\frac{fx}{2}] + b^2 d^2 x^2 \sin[\frac{fx}{2}] \right) \right) / \\
& \left(f (b+a \cos[e+fx])^2 \left(\cos[\frac{e}{2}] + \sin[\frac{e}{2}] \right) \left(\cos[\frac{e}{2} + \frac{fx}{2}] + \sin[\frac{e}{2} + \frac{fx}{2}] \right) \right) + \\
& \frac{1}{f^3 (b+a \cos[e+fx])^2} b \cos[e+fx]^2 (a+b \sec[e+fx])^2 \\
& (-2 i b c d f^2 x - i b d^2 f^2 x^2 - 4 i a c^2 f^2 \operatorname{ArcTan}[\cos[e+fx] + i \sin[e+fx]] - \\
& 8 i a c d f^2 x \operatorname{ArcTan}[\cos[e+fx] + i \sin[e+fx]] - 4 i a d^2 f^2 x^2 \\
& \operatorname{ArcTan}[\cos[e+fx] + i \sin[e+fx]] + 2 b c d f \operatorname{Log}[1+\cos[2(e+fx)] + i \sin[2(e+fx)]] + \\
& 2 b d^2 f x \operatorname{Log}[1+\cos[2(e+fx)] + i \sin[2(e+fx)]] - \\
& 4 i a d f (c+d x) \operatorname{PolyLog}[2, i \cos[e+fx] - \sin[e+fx]] + \\
& 4 i a d f (c+d x) \operatorname{PolyLog}[2, -i \cos[e+fx] + \sin[e+fx]] - i b d^2 \operatorname{PolyLog}[2, \\
& -\cos[2(e+fx)] - i \sin[2(e+fx)]] + 4 a d^2 \operatorname{PolyLog}[3, i \cos[e+fx] - \sin[e+fx]] - \\
& 4 a d^2 \operatorname{PolyLog}[3, -i \cos[e+fx] + \sin[e+fx]] + 2 b c d f^2 x \operatorname{Tan}[e] + b d^2 f^2 x^2 \operatorname{Tan}[e])
\end{aligned}$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int (c+dx) (a+b \sec[e+fx])^2 dx$$

Optimal (type 4, 131 leaves, 9 steps):

$$\begin{aligned}
& \frac{a^2 (c+dx)^2}{2d} - \frac{4 i a b (c+dx) \operatorname{ArcTan}[e^{i (e+fx)}]}{f} + \frac{b^2 d \operatorname{Log}[\cos[e+fx]]}{f^2} + \\
& \frac{2 i a b d \operatorname{PolyLog}[2, -i e^{i (e+fx)}]}{f^2} - \frac{2 i a b d \operatorname{PolyLog}[2, i e^{i (e+fx)}]}{f^2} + \frac{b^2 (c+dx) \operatorname{Tan}[e+fx]}{f}
\end{aligned}$$

Result (type 4, 554 leaves):

$$\begin{aligned}
& \frac{1}{2 f^2} \left(-a^2 (e + f x) (-2 c f + d (e - f x)) + \right. \\
& 2 b \left(-2 a d (e + f x) \left(\text{Log}[1 - \tan[\frac{1}{2} (e + f x)]] - \text{Log}[1 + \tan[\frac{1}{2} (e + f x)]] \right) + \right. \\
& a (d e - c f) \left(e + f x + \text{Log}[-1 + \tan[\frac{1}{2} (e + f x)]] - \text{Log}[1 + \tan[\frac{1}{2} (e + f x)]] \right) - \\
& a (d e - c f) \left(e + f x - \text{Log}[1 - \tan[\frac{1}{2} (e + f x)]] + \text{Log}[1 + \tan[\frac{1}{2} (e + f x)]] \right) + \\
& b d \left(-\text{Log}[\sec[\frac{1}{2} (e + f x)]^2] + \text{Log}[-1 + \tan[\frac{1}{2} (e + f x)]] + \text{Log}[4 \left(1 + \tan[\frac{1}{2} (e + f x)] \right)] \right) - \\
& 2 i a d \left(\text{Log}[1 - \tan[\frac{1}{2} (e + f x)]] \text{Log}[\left(\frac{1}{2} + \frac{i}{2}\right) \left(-i + \tan[\frac{1}{2} (e + f x)]\right)] - \right. \\
& \text{Log}[1 - \tan[\frac{1}{2} (e + f x)]] \text{Log}[\left(\frac{1}{2} - \frac{i}{2}\right) \left(i + \tan[\frac{1}{2} (e + f x)]\right)] - \\
& \text{Log}[\frac{1}{2} \left((1+i) - (1-i) \tan[\frac{1}{2} (e + f x)] \right)] \text{Log}[1 + \tan[\frac{1}{2} (e + f x)]] + \\
& \text{Log}[\left(-\frac{1}{2} - \frac{i}{2}\right) \left(i + \tan[\frac{1}{2} (e + f x)]\right)] \text{Log}[1 + \tan[\frac{1}{2} (e + f x)]] + \text{PolyLog}[2, \left(-\frac{1}{2} - \frac{i}{2}\right) \\
& \left(-1 + \tan[\frac{1}{2} (e + f x)]\right)] - \text{PolyLog}[2, \left(-\frac{1}{2} + \frac{i}{2}\right) \left(-1 + \tan[\frac{1}{2} (e + f x)]\right)] - \text{PolyLog}[\\
& 2, \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \tan[\frac{1}{2} (e + f x)]\right)] + \text{PolyLog}[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \tan[\frac{1}{2} (e + f x)]\right)] \right) + \\
& \left. \frac{2 b^2 f (c + d x) \sin[\frac{1}{2} (e + f x)]}{\cos[\frac{1}{2} (e + f x)] - \sin[\frac{1}{2} (e + f x)]} + \frac{2 b^2 f (c + d x) \sin[\frac{1}{2} (e + f x)]}{\cos[\frac{1}{2} (e + f x)] + \sin[\frac{1}{2} (e + f x)]} \right)
\end{aligned}$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^3}{a + b \sec[e + f x]} dx$$

Optimal (type 4, 526 leaves, 14 steps):

$$\begin{aligned}
& \frac{(c + d x)^4}{4 a d} + \frac{\frac{i b (c + d x)^3 \text{Log}[1 + \frac{a e^{i (e + f x)}}{b - \sqrt{-a^2 + b^2}}]}{a \sqrt{-a^2 + b^2} f} - \frac{\frac{i b (c + d x)^3 \text{Log}[1 + \frac{a e^{i (e + f x)}}{b + \sqrt{-a^2 + b^2}}]}{a \sqrt{-a^2 + b^2} f} + } \\
& \frac{3 b d (c + d x)^2 \text{PolyLog}[2, -\frac{a e^{i (e + f x)}}{b - \sqrt{-a^2 + b^2}}]}{a \sqrt{-a^2 + b^2} f^2} - \frac{3 b d (c + d x)^2 \text{PolyLog}[2, -\frac{a e^{i (e + f x)}}{b + \sqrt{-a^2 + b^2}}]}{a \sqrt{-a^2 + b^2} f^2} + \\
& \frac{6 \frac{i}{2} b d^2 (c + d x) \text{PolyLog}[3, -\frac{a e^{i (e + f x)}}{b - \sqrt{-a^2 + b^2}}]}{a \sqrt{-a^2 + b^2} f^3} - \frac{6 \frac{i}{2} b d^2 (c + d x) \text{PolyLog}[3, -\frac{a e^{i (e + f x)}}{b + \sqrt{-a^2 + b^2}}]}{a \sqrt{-a^2 + b^2} f^3} - \\
& \frac{6 b d^3 \text{PolyLog}[4, -\frac{a e^{i (e + f x)}}{b - \sqrt{-a^2 + b^2}}]}{a \sqrt{-a^2 + b^2} f^4} + \frac{6 b d^3 \text{PolyLog}[4, -\frac{a e^{i (e + f x)}}{b + \sqrt{-a^2 + b^2}}]}{a \sqrt{-a^2 + b^2} f^4}
\end{aligned}$$

Result (type 4, 1356 leaves) :

$$\begin{aligned}
& \frac{x (4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3) (b + a \cos[e + f x]) \sec[e + f x]}{4 a (a + b \sec[e + f x])} + \\
& \frac{1}{a \sqrt{a^2 - b^2} \sqrt{(-a^2 + b^2) e^{2 i e}} f^4 (a + b \sec[e + f x])} \\
& b (b + a \cos[e + f x]) \left(2 \frac{i}{2} c^3 \sqrt{(-a^2 + b^2) e^{2 i e}} f^3 \operatorname{ArcTan}\left[\frac{b + a e^{i (e+f x)}}{\sqrt{a^2 - b^2}}\right] + \right. \\
& 3 \frac{i}{2} \sqrt{a^2 - b^2} c^2 d e^{i e} f^3 x \log\left[1 + \frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] + \\
& 3 \frac{i}{2} \sqrt{a^2 - b^2} c d^2 e^{i e} f^3 x^2 \log\left[1 + \frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] + \\
& \frac{i}{2} \sqrt{a^2 - b^2} d^3 e^{i e} f^3 x^3 \log\left[1 + \frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - \\
& 3 \frac{i}{2} \sqrt{a^2 - b^2} c^2 d e^{i e} f^3 x \log\left[1 + \frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - \\
& 3 \frac{i}{2} \sqrt{a^2 - b^2} c d^2 e^{i e} f^3 x^2 \log\left[1 + \frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] - \\
& \frac{i}{2} \sqrt{a^2 - b^2} d^3 e^{i e} f^3 x^3 \log\left[1 + \frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}\right] + \\
& 3 \sqrt{a^2 - b^2} d e^{i e} f^2 (c + d x)^2 \operatorname{PolyLog}[2, -\frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}] - \\
& 3 \sqrt{a^2 - b^2} d e^{i e} f^2 (c + d x)^2 \operatorname{PolyLog}[2, -\frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}] + \\
& 6 \frac{i}{2} \sqrt{a^2 - b^2} c d^2 e^{i e} f \operatorname{PolyLog}[3, -\frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}] + \\
& 6 \frac{i}{2} \sqrt{a^2 - b^2} d^3 e^{i e} f x \operatorname{PolyLog}[3, -\frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}] - \\
& 6 \frac{i}{2} \sqrt{a^2 - b^2} c d^2 e^{i e} f \operatorname{PolyLog}[3, -\frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}] - \\
& 6 \frac{i}{2} \sqrt{a^2 - b^2} d^3 e^{i e} f x \operatorname{PolyLog}[3, -\frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}] - \\
& 6 \sqrt{a^2 - b^2} d^3 e^{i e} \operatorname{PolyLog}[4, -\frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}] +
\end{aligned}$$

$$\left. \frac{6 \sqrt{a^2 - b^2} d^3 e^{i e} \text{PolyLog}[4, -\frac{a e^{i (2 e + f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}] \right) \sec[e + f x]$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^3}{(a + b \sec[e + f x])^2} dx$$

Optimal (type 4, 1523 leaves, 36 steps):

$$\begin{aligned}
& -\frac{\frac{i}{2} b^2 (c+d x)^3}{a^2 (a^2-b^2) f} + \frac{(c+d x)^4}{4 a^2 d} + \frac{3 b^2 d (c+d x)^2 \operatorname{Log}\left[1+\frac{a e^{i(e+f x)}}{b-i \sqrt{a^2-b^2}}\right]}{a^2 (a^2-b^2) f^2} + \\
& \frac{3 b^2 d (c+d x)^2 \operatorname{Log}\left[1+\frac{a e^{i(e+f x)}}{b+i \sqrt{a^2-b^2}}\right]}{a^2 (a^2-b^2) f^2} - \frac{i b^3 (c+d x)^3 \operatorname{Log}\left[1+\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} f} + \\
& \frac{2 i b (c+d x)^3 \operatorname{Log}\left[1+\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} f} + \frac{i b^3 (c+d x)^3 \operatorname{Log}\left[1+\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} f} - \\
& \frac{2 i b (c+d x)^3 \operatorname{Log}\left[1+\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} f} - \frac{6 i b^2 d^2 (c+d x) \operatorname{PolyLog}[2, -\frac{a e^{i(e+f x)}}{b-i \sqrt{a^2-b^2}}]}{a^2 (a^2-b^2) f^3} - \\
& \frac{6 i b^2 d^2 (c+d x) \operatorname{PolyLog}[2, -\frac{a e^{i(e+f x)}}{b+i \sqrt{a^2-b^2}}]}{a^2 (a^2-b^2) f^3} - \frac{3 b^3 d (c+d x)^2 \operatorname{PolyLog}[2, -\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}]}{a^2 (-a^2+b^2)^{3/2} f^2} + \\
& \frac{6 b d (c+d x)^2 \operatorname{PolyLog}[2, -\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}]}{a^2 \sqrt{-a^2+b^2} f^2} + \frac{3 b^3 d (c+d x)^2 \operatorname{PolyLog}[2, -\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}]}{a^2 (-a^2+b^2)^{3/2} f^2} - \\
& \frac{6 b d (c+d x)^2 \operatorname{PolyLog}[2, -\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}]}{a^2 \sqrt{-a^2+b^2} f^2} + \frac{6 b^2 d^3 \operatorname{PolyLog}[3, -\frac{a e^{i(e+f x)}}{b-i \sqrt{a^2-b^2}}]}{a^2 (a^2-b^2) f^4} + \\
& \frac{6 b^2 d^3 \operatorname{PolyLog}[3, -\frac{a e^{i(e+f x)}}{b+i \sqrt{a^2-b^2}}]}{a^2 (a^2-b^2) f^4} - \frac{6 i b^3 d^2 (c+d x) \operatorname{PolyLog}[3, -\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}]}{a^2 (-a^2+b^2)^{3/2} f^3} + \\
& \frac{12 i b d^2 (c+d x) \operatorname{PolyLog}[3, -\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}]}{a^2 \sqrt{-a^2+b^2} f^3} + \frac{6 i b^3 d^2 (c+d x) \operatorname{PolyLog}[3, -\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}]}{a^2 (-a^2+b^2)^{3/2} f^3} - \\
& \frac{12 i b d^2 (c+d x) \operatorname{PolyLog}[3, -\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}]}{a^2 \sqrt{-a^2+b^2} f^3} + \frac{6 b^3 d^3 \operatorname{PolyLog}[4, -\frac{a e^{i(e+f x)}}{b-i \sqrt{-a^2+b^2}}]}{a^2 (-a^2+b^2)^{3/2} f^4} - \\
& \frac{12 b d^3 \operatorname{PolyLog}[4, -\frac{a e^{i(e+f x)}}{b-\sqrt{-a^2+b^2}}]}{a^2 \sqrt{-a^2+b^2} f^4} - \frac{6 b^3 d^3 \operatorname{PolyLog}[4, -\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}]}{a^2 (-a^2+b^2)^{3/2} f^4} + \\
& \frac{12 b d^3 \operatorname{PolyLog}[4, -\frac{a e^{i(e+f x)}}{b+\sqrt{-a^2+b^2}}]}{a^2 \sqrt{-a^2+b^2} f^4} + \frac{b^2 (c+d x)^3 \operatorname{Sin}[e+f x]}{a (a^2-b^2) f (b+a \operatorname{Cos}[e+f x])}
\end{aligned}$$

Result (type 4, 9003 leaves):

$$-\frac{1}{(a^2-b^2)^{3/2} f^2 (a+b \operatorname{Sec}[e+f x])^2}$$

$$\begin{aligned}
& 6 b c^2 d (b + a \cos[e + f x])^2 \left(2 (e + f x) \operatorname{ArcTanh} \left[\frac{(a + b) \cot[\frac{1}{2} (e + f x)]}{\sqrt{a^2 - b^2}} \right] - \right. \\
& 2 \left(e + \operatorname{ArcCos} \left[-\frac{b}{a} \right] \right) \operatorname{ArcTanh} \left[\frac{(a - b) \tan[\frac{1}{2} (e + f x)]}{\sqrt{a^2 - b^2}} \right] + \left(\operatorname{ArcCos} \left[-\frac{b}{a} \right] - \right. \\
& 2 \left. \left. \left. \operatorname{ArcTanh} \left[\frac{(a + b) \cot[\frac{1}{2} (e + f x)]}{\sqrt{a^2 - b^2}} \right] - \operatorname{ArcTanh} \left[\frac{(a - b) \tan[\frac{1}{2} (e + f x)]}{\sqrt{a^2 - b^2}} \right] \right) \right) \\
& \operatorname{Log} \left[\frac{\sqrt{a^2 - b^2} e^{-\frac{1}{2} i (e + f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \cos[e + f x]}} \right] + \left(\operatorname{ArcCos} \left[-\frac{b}{a} \right] + \right. \\
& 2 \left. \left. \left. \operatorname{ArcTanh} \left[\frac{(a + b) \cot[\frac{1}{2} (e + f x)]}{\sqrt{a^2 - b^2}} \right] - \operatorname{ArcTanh} \left[\frac{(a - b) \tan[\frac{1}{2} (e + f x)]}{\sqrt{a^2 - b^2}} \right] \right) \right) \\
& \operatorname{Log} \left[\frac{\sqrt{a^2 - b^2} e^{\frac{1}{2} i (e + f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \cos[e + f x]}} \right] - \left(\operatorname{ArcCos} \left[-\frac{b}{a} \right] + 2 \operatorname{ArcTanh} \left[\frac{(a - b) \tan[\frac{1}{2} (e + f x)]}{\sqrt{a^2 - b^2}} \right] \right) \\
& \operatorname{Log} \left[1 - \frac{\left(b - \frac{1}{2} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e + f x)] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e + f x)] \right)} \right] + \\
& \left(-\operatorname{ArcCos} \left[-\frac{b}{a} \right] + 2 \operatorname{ArcTanh} \left[\frac{(a - b) \tan[\frac{1}{2} (e + f x)]}{\sqrt{a^2 - b^2}} \right] \right) \\
& \operatorname{Log} \left[1 - \frac{\left(b + \frac{1}{2} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e + f x)] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e + f x)] \right)} \right] + \\
& \left. \left. \left. \frac{1}{\operatorname{PolyLog}[2, \frac{\left(b - \frac{1}{2} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e + f x)] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e + f x)] \right)}]} - \right. \right. \right. \\
& \left. \left. \left. \operatorname{PolyLog}[2, \frac{\left(b + \frac{1}{2} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e + f x)] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e + f x)] \right)}] \right) \right) \operatorname{Sec}[e + f x]^2 + \right. \\
& \frac{1}{a^2 (a^2 - b^2)^{3/2} f^2 (a + b \operatorname{Sec}[e + f x])^2} 3 b^3 c^2 d (b + a \cos[e + f x])^2 \\
& \left(2 (e + f x) \operatorname{ArcTanh} \left[\frac{(a + b) \cot[\frac{1}{2} (e + f x)]}{\sqrt{a^2 - b^2}} \right] - \right. \\
& 2 \left(e + \operatorname{ArcCos} \left[-\frac{b}{a} \right] \right) \operatorname{ArcTanh} \left[\frac{(a - b) \tan[\frac{1}{2} (e + f x)]}{\sqrt{a^2 - b^2}} \right] + \\
& \left. \left. \left. \left(\operatorname{ArcCos} \left[-\frac{b}{a} \right] - 2 \operatorname{ArcTanh} \left[\frac{(a + b) \cot[\frac{1}{2} (e + f x)]}{\sqrt{a^2 - b^2}} \right] - \operatorname{ArcTanh} \left[\frac{(a - b) \tan[\frac{1}{2} (e + f x)]}{\sqrt{a^2 - b^2}} \right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Log} \left[\frac{\sqrt{a^2 - b^2} e^{-\frac{1}{2} i (e+f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \cos[e + f x]}} \right] + \\
& \left(\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \text{i} \left(\text{ArcTanh} \left[\frac{(a+b) \cot[\frac{1}{2} (e+f x)]}{\sqrt{a^2 - b^2}} \right] - \text{ArcTanh} \left[\frac{(a-b) \tan[\frac{1}{2} (e+f x)]}{\sqrt{a^2 - b^2}} \right] \right) \right) \\
& \text{Log} \left[\frac{\sqrt{a^2 - b^2} e^{\frac{1}{2} i (e+f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \cos[e + f x]}} \right] - \left(\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \text{i} \text{ArcTanh} \left[\frac{(a-b) \tan[\frac{1}{2} (e+f x)]}{\sqrt{a^2 - b^2}} \right] \right) \\
& \text{Log} \left[1 - \frac{\left(b - \text{i} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e+f x)] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e+f x)] \right)} \right] + \\
& \left(-\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \text{i} \text{ArcTanh} \left[\frac{(a-b) \tan[\frac{1}{2} (e+f x)]}{\sqrt{a^2 - b^2}} \right] \right) \\
& \text{Log} \left[1 - \frac{\left(b + \text{i} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e+f x)] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e+f x)] \right)} \right] + \\
& \text{i} \left(\text{PolyLog}[2, \frac{\left(b - \text{i} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e+f x)] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e+f x)] \right)}] - \right. \\
& \left. \text{PolyLog}[2, \frac{\left(b + \text{i} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e+f x)] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \tan[\frac{1}{2} (e+f x)] \right)}] \right) \text{Sec}[e + f x]^2 - \\
& \left(6 b c d^2 e^{i e} (b + a \cos[e + f x])^2 \left(-2 f x \text{PolyLog}[2, -\frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}] - \right. \right. \\
& \left. \left. \text{i} \left(f^2 x^2 \text{Log} \left[1 + \frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] - f^2 x^2 \text{Log} \left[1 + \frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] + \right. \right. \\
& \left. \left. 2 \text{i} f x \text{PolyLog}[2, -\frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}] + \right. \right. \\
& \left. \left. 2 \text{PolyLog}[3, -\frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}] - 2 \text{PolyLog}[3, -\frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}] \right) \right) \\
& \text{Sec}[e + f x]^2 \Bigg) / \left((a^2 - b^2) \sqrt{(-a^2 + b^2) e^{2 i e}} f^3 (a + b \text{Sec}[e + f x])^2 \right) + \\
& \left(3 b^3 c d^2 e^{i e} (b + a \cos[e + f x])^2 \left(-2 f x \text{PolyLog}[2, -\frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{(a^2 - b^2) \sqrt{(-a^2 + b^2) e^{2i\theta}} f^4 (a + b \sec(e + f x))^2} \\
& \left(\frac{1}{2} \left(\frac{f^2 x^2 \operatorname{Log} \left[1 + \frac{a e^{i(2e+f x)}}{b e^{i\theta} - \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] - f^2 x^2 \operatorname{Log} \left[1 + \frac{a e^{i(2e+f x)}}{b e^{i\theta} + \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] + \right. \right. \\
& \quad 2 i f x \operatorname{PolyLog} \left[2, - \frac{a e^{i(2e+f x)}}{b e^{i\theta} + \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] + \\
& \quad 2 \operatorname{PolyLog} \left[3, - \frac{a e^{i(2e+f x)}}{b e^{i\theta} - \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] - 2 \operatorname{PolyLog} \left[3, - \frac{a e^{i(2e+f x)}}{b e^{i\theta} + \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] \left. \right) \\
& \left. \operatorname{Sec}(e + f x)^2 \right) / \left(a^2 (a^2 - b^2) \sqrt{(-a^2 + b^2) e^{2i\theta}} f^3 (a + b \operatorname{Sec}(e + f x))^2 \right) - \\
& \frac{1}{(a^2 - b^2) \sqrt{(-a^2 + b^2) e^{2i\theta}} f^4 (a + b \operatorname{Sec}(e + f x))^2} \\
& \frac{b}{d^3} \\
& \frac{e^{i\theta}}{(b + a \cos(e + f x))^2} \\
& \left(- \frac{i f^3 x^3 \operatorname{Log} \left[1 + \frac{a e^{i(2e+f x)}}{b e^{i\theta} - \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] + i f^3 x^3 \operatorname{Log} \left[1 + \frac{a e^{i(2e+f x)}}{b e^{i\theta} + \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] - \right. \right. \\
& \quad 3 f^2 x^2 \operatorname{PolyLog} \left[2, - \frac{a e^{i(2e+f x)}}{b e^{i\theta} - \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] + \\
& \quad 3 f^2 x^2 \operatorname{PolyLog} \left[2, - \frac{a e^{i(2e+f x)}}{b e^{i\theta} + \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] - \\
& \quad 6 i f x \operatorname{PolyLog} \left[3, - \frac{a e^{i(2e+f x)}}{b e^{i\theta} - \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] + \\
& \quad 6 i f x \operatorname{PolyLog} \left[3, - \frac{a e^{i(2e+f x)}}{b e^{i\theta} + \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] + 6 \operatorname{PolyLog} \left[4, - \frac{a e^{i(2e+f x)}}{b e^{i\theta} - \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] - \\
& \quad \left. \left. 6 \operatorname{PolyLog} \left[4, - \frac{a e^{i(2e+f x)}}{b e^{i\theta} + \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] \right) \operatorname{Sec}(e + f x)^2 + \right. \\
& \frac{1}{a^2 (a^2 - b^2) \sqrt{(-a^2 + b^2) e^{2i\theta}} f^4 (a + b \operatorname{Sec}(e + f x))^2} \\
& \frac{b^3 d^3 e^{i\theta}}{(b + a \cos(e + f x))^2} \\
& \left(- \frac{i f^3 x^3 \operatorname{Log} \left[1 + \frac{a e^{i(2e+f x)}}{b e^{i\theta} - \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] + i f^3 x^3 \operatorname{Log} \left[1 + \frac{a e^{i(2e+f x)}}{b e^{i\theta} + \sqrt{(-a^2 + b^2) e^{2i\theta}}} \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{3 f^2 x^2 \operatorname{PolyLog}[2, -\frac{a e^{i(2e+fx)}}{b e^{i e} - \sqrt{(-a^2 + b^2)} e^{2i e}}] + \\
& 3 f^2 x^2 \operatorname{PolyLog}[2, -\frac{a e^{i(2e+fx)}}{b e^{i e} + \sqrt{(-a^2 + b^2)} e^{2i e}}] - \\
& 6 i f x \operatorname{PolyLog}[3, -\frac{a e^{i(2e+fx)}}{b e^{i e} - \sqrt{(-a^2 + b^2)} e^{2i e}}] + \\
& 6 i f x \operatorname{PolyLog}[3, -\frac{a e^{i(2e+fx)}}{b e^{i e} + \sqrt{(-a^2 + b^2)} e^{2i e}}] + 6 \operatorname{PolyLog}[4, -\frac{a e^{i(2e+fx)}}{b e^{i e} - \sqrt{(-a^2 + b^2)} e^{2i e}}] - \\
& 6 \operatorname{PolyLog}[4, -\frac{a e^{i(2e+fx)}}{b e^{i e} + \sqrt{(-a^2 + b^2)} e^{2i e}}] \operatorname{Sec}[e + f x]^2 - \\
& \frac{1}{2 a^2 (a^2 - b^2) \sqrt{(-a^2 + b^2)} e^{2i e} f^4 (a + b \operatorname{Sec}[e + f x])^2} \\
& b^2 d^3 e^{-i e} (b + a \operatorname{Cos}[e + f x])^2 \\
& \left(2 i e^{2i e} \sqrt{(-a^2 + b^2)} e^{2i e} f^3 x^3 + 3 b e^{i e} f^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{i(2e+fx)}}{b e^{i e} - \sqrt{(-a^2 + b^2)} e^{2i e}}\right] - \right. \\
& 3 b e^{3i e} f^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{i(2e+fx)}}{b e^{i e} - \sqrt{(-a^2 + b^2)} e^{2i e}}\right] - \\
& 3 \sqrt{(-a^2 + b^2)} e^{2i e} f^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{i(2e+fx)}}{b e^{i e} - \sqrt{(-a^2 + b^2)} e^{2i e}}\right] - \\
& 3 e^{2i e} \sqrt{(-a^2 + b^2)} e^{2i e} f^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{i(2e+fx)}}{b e^{i e} - \sqrt{(-a^2 + b^2)} e^{2i e}}\right] - \\
& 3 b e^{i e} f^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{i(2e+fx)}}{b e^{i e} + \sqrt{(-a^2 + b^2)} e^{2i e}}\right] + \\
& 3 b e^{3i e} f^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{i(2e+fx)}}{b e^{i e} + \sqrt{(-a^2 + b^2)} e^{2i e}}\right] - \\
& 3 \sqrt{(-a^2 + b^2)} e^{2i e} f^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{i(2e+fx)}}{b e^{i e} + \sqrt{(-a^2 + b^2)} e^{2i e}}\right] - \\
& 3 e^{2i e} \sqrt{(-a^2 + b^2)} e^{2i e} f^2 x^2 \operatorname{Log}\left[1 + \frac{a e^{i(2e+fx)}}{b e^{i e} + \sqrt{(-a^2 + b^2)} e^{2i e}}\right] + \\
& 6 i \left(b e^{i e} (-1 + e^{2i e}) + \sqrt{(-a^2 + b^2)} e^{2i e} (1 + e^{2i e}) \right) f x \\
& \operatorname{PolyLog}[2, -\frac{a e^{i(2e+fx)}}{b e^{i e} - \sqrt{(-a^2 + b^2)} e^{2i e}}] + 6 i \left(\sqrt{(-a^2 + b^2)} e^{2i e} (1 + e^{2i e}) + b (e^{i e} - e^{3i e}) \right)
\end{aligned}$$

$$\begin{aligned}
& f \times \text{PolyLog}[2, -\frac{a e^{i(2e+fx)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}] + 6 b e^{i e} \\
& \text{PolyLog}[3, -\frac{a e^{i(2e+fx)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}] - 6 b e^{3 i e} \text{PolyLog}[3, -\frac{a e^{i(2e+fx)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}] - \\
& 6 \sqrt{(-a^2 + b^2) e^{2 i e}} \text{PolyLog}[3, -\frac{a e^{i(2e+fx)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}] - \\
& 6 e^{2 i e} \sqrt{(-a^2 + b^2) e^{2 i e}} \text{PolyLog}[3, -\frac{a e^{i(2e+fx)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}}] - \\
& 6 b e^{i e} \text{PolyLog}[3, -\frac{a e^{i(2e+fx)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}] + \\
& 6 b e^{3 i e} \text{PolyLog}[3, -\frac{a e^{i(2e+fx)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}] - \\
& 6 \sqrt{(-a^2 + b^2) e^{2 i e}} \text{PolyLog}[3, -\frac{a e^{i(2e+fx)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}] - \\
& 6 e^{2 i e} \sqrt{(-a^2 + b^2) e^{2 i e}} \text{PolyLog}[3, -\frac{a e^{i(2e+fx)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}}] \Bigg) \sec[e] \sec[e + fx]^2 - \\
& \left(4 \frac{i b c^3}{a} \text{ArcTan}\left[\frac{-\frac{i}{a} a \sin[e] - \frac{i}{a} (-b + a \cos[e]) \tan\left[\frac{fx}{2}\right]}{\sqrt{-b^2 + a^2 \cos[e]^2 + a^2 \sin[e]^2}}\right] (b + a \cos[e + fx])^2 \sec[e + fx]^2 \right) / \\
& \left((a^2 - b^2) f (a + b \sec[e + fx])^2 \sqrt{-b^2 + a^2 \cos[e]^2 + a^2 \sin[e]^2} \right) + \\
& \left(2 \frac{i b^3 c^3}{a} \text{ArcTan}\left[\frac{-\frac{i}{a} a \sin[e] - \frac{i}{a} (-b + a \cos[e]) \tan\left[\frac{fx}{2}\right]}{\sqrt{-b^2 + a^2 \cos[e]^2 + a^2 \sin[e]^2}}\right] (b + a \cos[e + fx])^2 \sec[e + fx]^2 \right) / \\
& \left(a^2 (a^2 - b^2) f (a + b \sec[e + fx])^2 \sqrt{-b^2 + a^2 \cos[e]^2 + a^2 \sin[e]^2} \right) + \\
& \left(3 b^2 c^2 d (b + a \cos[e + fx])^2 \sec[e] \sec[e + fx]^2 \right. \\
& \left. a \cos[e] \log[b + a \cos[e] \cos[fx] - a \sin[e] \sin[fx]] + \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{2 i a b \operatorname{ArcTan} \left[\frac{-i a \sin[e] - i (-b + a \cos[e]) \tan \left[\frac{f x}{2} \right]}{\sqrt{-b^2 + a^2 \cos[e]^2 + a^2 \sin[e]^2}} \right] \sin[e]}{a f x \sin[e]} \right) \right\} / \\
& \frac{\left(a (a^2 - b^2) f^2 (a + b \sec[e + f x])^2 (a^2 \cos[e]^2 + a^2 \sin[e]^2) \right) - \\
& \frac{1}{a (a^2 - b^2) f (a + b \sec[e + f x])^2} \\
& 6 b^2 c d^2 (b + a \cos[e + f x])^2 \sec[e] \sec[e + f x]^2 \\
& \left(\frac{x^2 \sin[e]}{2 a} - \frac{1}{a f} x \left(\cos[e] \log[b + a \cos[e + f x]] + f x \sin[e] + \right. \right. \\
& \left. \left. b \operatorname{ArcTan} \left[\left(i \cos[e] + \sin[e] \right) \left(a \sin[e] + (-b + a \cos[e]) \tan \left[\frac{f x}{2} \right] \right) \right] \right) / \right. \\
& \left. \left(\sqrt{a^2 - b^2} \sqrt{(\cos[e] - i \sin[e])^2} \right) \right) (2 \sin[e]^2 + i \sin[2 e]) / \\
& \left(\sqrt{a^2 - b^2} \sqrt{(\cos[e] - i \sin[e])^2} \right) + \frac{1}{a f} \left(\frac{(e + f x) \cos[e] \log[b + a \cos[e + f x]]}{f} + \right. \\
& \left. \frac{1}{a} \cos[e] \left(-\frac{(e + f x) \log[b + a \cos[e + f x]]}{a} + \frac{1}{a} \left(\frac{1}{2} i (e + f x)^2 - 4 i \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{a}}}{\sqrt{2}} \right] \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcTan} \left[\frac{(-a + b) \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-a^2 + b^2}} \right] - \left(e + f x + 2 \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{a}}}{\sqrt{2}} \right] \right) \right) \right) \right. \\
& \left. \left. \operatorname{Log} \left[1 + \frac{\left(b - \sqrt{-a^2 + b^2} \right) e^{i (e + f x)}}{a} \right] - \left(e + f x - 2 \operatorname{ArcSin} \left[\frac{\sqrt{\frac{a+b}{a}}}{\sqrt{2}} \right] \right) \right. \\
& \left. \left. \operatorname{Log} \left[1 + \frac{\left(b + \sqrt{-a^2 + b^2} \right) e^{i (e + f x)}}{a} \right] + (e + f x) \log[b + a \cos[e + f x]] + i \left(\operatorname{PolyLog} \left[\right. \right. \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\text{Cos}[e] + \text{Sin}[e]}{\text{Cos}[e]^2 + \text{Sin}[2e]} \right) \right\} + \frac{1}{8 a^2 (a^2 - b^2) f (a + b \text{Sec}[e + f x])^2} \\
& (b + a \text{Cos}[e + f x]) \text{Sec}[e] \text{Sec}[e + f x]^2 (8 a^2 b c^3 f x \text{Cos}[e] - 8 b^3 c^3 f x \text{Cos}[e] + \\
& 12 a^2 b c^2 d f x^2 \text{Cos}[e] - 12 b^3 c^2 d f x^2 \text{Cos}[e] + 8 a^2 b c d^2 f x^3 \text{Cos}[e] - \\
& 8 b^3 c d^2 f x^3 \text{Cos}[e] + 2 a^2 b d^3 f x^4 \text{Cos}[e] - 2 b^3 d^3 f x^4 \text{Cos}[e] + \\
& 4 a^3 c^3 f x \text{Cos}[f x] - 4 a b^2 c^3 f x \text{Cos}[f x] + 6 a^3 c^2 d f x^2 \text{Cos}[f x] - \\
& 6 a b^2 c^2 d f x^2 \text{Cos}[f x] + 4 a^3 c d^2 f x^3 \text{Cos}[f x] - 4 a b^2 c d^2 f x^3 \text{Cos}[f x] + \\
& a^3 d^3 f x^4 \text{Cos}[f x] - a b^2 d^3 f x^4 \text{Cos}[f x] + 4 a^3 c^3 f x \text{Cos}[2e + f x] - \\
& 4 a b^2 c^3 f x \text{Cos}[2e + f x] + 6 a^3 c^2 d f x^2 \text{Cos}[2e + f x] - \\
& 6 a b^2 c^2 d f x^2 \text{Cos}[2e + f x] + 4 a^3 c d^2 f x^3 \text{Cos}[2e + f x] - \\
& 4 a b^2 c d^2 f x^3 \text{Cos}[2e + f x] + a^3 d^3 f x^4 \text{Cos}[2e + f x] - \\
& a b^2 d^3 f x^4 \text{Cos}[2e + f x] - 8 b^3 c^3 \text{Sin}[e] - 24 b^3 c^2 d x \text{Sin}[e] - \\
& 24 b^3 c d^2 x^2 \text{Sin}[e] - 8 b^3 d^3 x^3 \text{Sin}[e] + 8 a b^2 c^3 \text{Sin}[f x] + \\
& 24 a b^2 c^2 d x \text{Sin}[f x] + 24 a b^2 c d^2 x^2 \text{Sin}[f x] + 8 a b^2 d^3 x^3 \text{Sin}[f x]) + \\
& \frac{1}{a^2 (a^2 - b^2)^{3/2} f^3 (a + b \text{Sec}[e + f x])^2} \\
& \frac{6}{b^3} \\
& \frac{c}{d^2} \\
& (b + a \text{Cos}[e + f x])^2 \\
& \left(2 (e + f x) \text{ArcTanh} \left[\frac{(a + b) \text{Cot} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{a^2 - b^2}} \right] - \right. \\
& 2 \left(e + \text{ArcCos} \left[-\frac{b}{a} \right] \right) \text{ArcTanh} \left[\frac{(a - b) \text{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{a^2 - b^2}} \right] + \\
& \left. \left(\text{ArcCos} \left[-\frac{b}{a} \right] - 2 \frac{i}{2} \left(\text{ArcTanh} \left[\frac{(a + b) \text{Cot} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{a^2 - b^2}} \right] - \text{ArcTanh} \left[\frac{(a - b) \text{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{a^2 - b^2}} \right] \right) \right) \right. \\
& \text{Log} \left[\frac{\sqrt{a^2 - b^2} e^{-\frac{1}{2} i (e + f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \text{Cos}[e + f x]}} \right] + \\
& \left(\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \frac{i}{2} \left(\text{ArcTanh} \left[\frac{(a + b) \text{Cot} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{a^2 - b^2}} \right] - \text{ArcTanh} \left[\frac{(a - b) \text{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{a^2 - b^2}} \right] \right) \right) \\
& \text{Log} \left[\frac{\sqrt{a^2 - b^2} e^{\frac{1}{2} i (e + f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \text{Cos}[e + f x]}} \right] - \left(\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \frac{i}{2} \text{ArcTanh} \left[\frac{(a - b) \text{Tan} \left[\frac{1}{2} (e + f x) \right]}{\sqrt{a^2 - b^2}} \right] \right) \\
& \text{Log} \left[1 - \frac{\left(b - \frac{i}{2} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \text{Tan} \left[\frac{1}{2} (e + f x) \right] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \text{Tan} \left[\frac{1}{2} (e + f x) \right] \right)} \right] +
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{ArcCos}\left[-\frac{b}{a} \right] + 2 \text{i} \text{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2-b^2}} \right] \right) \\
& \text{Log}\left[1 - \frac{\left(b + \text{i} \sqrt{a^2-b^2} \right) \left(a + b - \sqrt{a^2-b^2} \tan\left[\frac{1}{2} (e+f x) \right] \right)}{a \left(a + b + \sqrt{a^2-b^2} \tan\left[\frac{1}{2} (e+f x) \right] \right)} \right] + \\
& \text{i} \left(\text{PolyLog}\left[2, \frac{\left(b - \text{i} \sqrt{a^2-b^2} \right) \left(a + b - \sqrt{a^2-b^2} \tan\left[\frac{1}{2} (e+f x) \right] \right)}{a \left(a + b + \sqrt{a^2-b^2} \tan\left[\frac{1}{2} (e+f x) \right] \right)} \right] - \right. \\
& \left. \text{PolyLog}\left[2, \frac{\left(b + \text{i} \sqrt{a^2-b^2} \right) \left(a + b - \sqrt{a^2-b^2} \tan\left[\frac{1}{2} (e+f x) \right] \right)}{a \left(a + b + \sqrt{a^2-b^2} \tan\left[\frac{1}{2} (e+f x) \right] \right)} \right] \right) \\
& \text{Sec}[e+f x]^2 \tan[e] + \frac{1}{a^2 (a^2-b^2) \sqrt{(-a^2+b^2) e^{2 \text{i} e}} f^4 (a+b \text{Sec}[e+f x])^2} 3 \\
& b^3 \\
& d^3 \\
& e^{\text{i} e} \\
& (b+a \cos[e+f x])^2 \\
& \left(-2 f x \text{PolyLog}\left[2, -\frac{a e^{\text{i} (2 e+f x)}}{b e^{\text{i} e} - \sqrt{(-a^2+b^2) e^{2 \text{i} e}}} \right] - \right. \\
& \left. \text{i} \left(f^2 x^2 \text{Log}\left[1 + \frac{a e^{\text{i} (2 e+f x)}}{b e^{\text{i} e} - \sqrt{(-a^2+b^2) e^{2 \text{i} e}}} \right] - f^2 x^2 \text{Log}\left[1 + \frac{a e^{\text{i} (2 e+f x)}}{b e^{\text{i} e} + \sqrt{(-a^2+b^2) e^{2 \text{i} e}}} \right] + 2 \text{i} f x \right. \right. \\
& \left. \text{PolyLog}\left[2, -\frac{a e^{\text{i} (2 e+f x)}}{b e^{\text{i} e} + \sqrt{(-a^2+b^2) e^{2 \text{i} e}}} \right] + 2 \text{PolyLog}\left[3, -\frac{a e^{\text{i} (2 e+f x)}}{b e^{\text{i} e} - \sqrt{(-a^2+b^2) e^{2 \text{i} e}}} \right] - \right. \\
& \left. 2 \text{PolyLog}\left[3, -\frac{a e^{\text{i} (2 e+f x)}}{b e^{\text{i} e} + \sqrt{(-a^2+b^2) e^{2 \text{i} e}}} \right] \right) \text{Sec}[e+f x]^2 \tan[e] + \\
& \left(6 \text{i} b^3 c^2 d \text{ArcTan}\left[\frac{-\text{i} a \sin[e] - \text{i} (-b+a \cos[e]) \tan\left[\frac{f x}{2} \right]}{\sqrt{-b^2+a^2 \cos[e]^2+a^2 \sin[e]^2}} \right] (b+a \cos[e+f x])^2 \right. \\
& \left. \text{Sec}[e+f x]^2 \tan[e] \right) / \\
& \left(a^2 (a^2-b^2) f^2 (a+b \text{Sec}[e+f x])^2 \sqrt{-b^2+a^2 \cos[e]^2+a^2 \sin[e]^2} \right)
\end{aligned}$$

Problem 40: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+d x)^2}{(a+b \text{Sec}[e+f x])^2} dx$$

Optimal (type 4, 1117 leaves, 30 steps):

$$\begin{aligned}
& -\frac{\frac{1}{2} b^2 (c+d x)^2}{a^2 (a^2-b^2) f} + \frac{(c+d x)^3}{3 a^2 d} + \frac{2 b^2 d (c+d x) \operatorname{Log}\left[1+\frac{a e^{i (e+f x)}}{b-i \sqrt{a^2-b^2}}\right]}{a^2 (a^2-b^2) f^2} + \\
& \frac{2 b^2 d (c+d x) \operatorname{Log}\left[1+\frac{a e^{i (e+f x)}}{b+i \sqrt{a^2-b^2}}\right]}{a^2 (a^2-b^2) f^2} - \frac{\frac{1}{2} b^3 (c+d x)^2 \operatorname{Log}\left[1+\frac{a e^{i (e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} f} + \\
& \frac{2 \frac{1}{2} b (c+d x)^2 \operatorname{Log}\left[1+\frac{a e^{i (e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} f} + \frac{\frac{1}{2} b^3 (c+d x)^2 \operatorname{Log}\left[1+\frac{a e^{i (e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} f} - \\
& \frac{2 \frac{1}{2} b (c+d x)^2 \operatorname{Log}\left[1+\frac{a e^{i (e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} f} - \frac{2 \frac{1}{2} b^2 d^2 \operatorname{PolyLog}\left[2,-\frac{a e^{i (e+f x)}}{b-i \sqrt{a^2-b^2}}\right]}{a^2 (a^2-b^2) f^3} - \\
& \frac{2 \frac{1}{2} b^2 d^2 \operatorname{PolyLog}\left[2,-\frac{a e^{i (e+f x)}}{b+i \sqrt{a^2-b^2}}\right]}{a^2 (a^2-b^2) f^3} - \frac{2 b^3 d (c+d x) \operatorname{PolyLog}\left[2,-\frac{a e^{i (e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} f^2} + \\
& \frac{4 b d (c+d x) \operatorname{PolyLog}\left[2,-\frac{a e^{i (e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} f^2} + \frac{2 b^3 d (c+d x) \operatorname{PolyLog}\left[2,-\frac{a e^{i (e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} f^2} - \\
& \frac{4 b d (c+d x) \operatorname{PolyLog}\left[2,-\frac{a e^{i (e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} f^2} - \frac{2 \frac{1}{2} b^3 d^2 \operatorname{PolyLog}\left[3,-\frac{a e^{i (e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} f^3} + \\
& \frac{4 \frac{1}{2} b d^2 \operatorname{PolyLog}\left[3,-\frac{a e^{i (e+f x)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} f^3} + \frac{2 \frac{1}{2} b^3 d^2 \operatorname{PolyLog}\left[3,-\frac{a e^{i (e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 (-a^2+b^2)^{3/2} f^3} - \\
& \frac{4 \frac{1}{2} b d^2 \operatorname{PolyLog}\left[3,-\frac{a e^{i (e+f x)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2+b^2} f^3} + \frac{b^2 (c+d x)^2 \operatorname{Sin}[e+f x]}{a (a^2-b^2) f (b+a \operatorname{Cos}[e+f x])}
\end{aligned}$$

Result (type 4, 5576 leaves):

$$\begin{aligned}
& \frac{x (3 c^2 + 3 c d x + d^2 x^2) (b + a \operatorname{Cos}[e+f x])^2 \operatorname{Sec}[e+f x]^2}{3 a^2 (a + b \operatorname{Sec}[e+f x])^2} - \frac{1}{(a^2-b^2)^{3/2} f^2 (a + b \operatorname{Sec}[e+f x])^2} \\
& 4 b c d (b + a \operatorname{Cos}[e+f x])^2 \left(2 (e+f x) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2} (e+f x)\right]}{\sqrt{a^2-b^2}}\right] - \right. \\
& \left. 2 \left(e + \operatorname{ArcCos}\left[-\frac{b}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]}{\sqrt{a^2-b^2}}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 \frac{1}{2} \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2} (e+f x)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \text{Log} \left[\frac{\sqrt{a^2 - b^2} e^{-\frac{1}{2} i (e+f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \cos[e + f x]}} \right] + \\
& \left(\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \text{i} \left(\text{ArcTanh} \left[\frac{(a+b) \text{Cot} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2 - b^2}} \right] - \text{ArcTanh} \left[\frac{(a-b) \text{Tan} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2 - b^2}} \right] \right) \right) \\
& \text{Log} \left[\frac{\sqrt{a^2 - b^2} e^{\frac{1}{2} i (e+f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \cos[e + f x]}} \right] - \left(\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \text{i} \text{ArcTanh} \left[\frac{(a-b) \text{Tan} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2 - b^2}} \right] \right) \\
& \text{Log} \left[1 - \frac{\left(b - \text{i} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \text{Tan} \left[\frac{1}{2} (e+f x) \right] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \text{Tan} \left[\frac{1}{2} (e+f x) \right] \right)} \right] + \\
& \left(-\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \text{i} \text{ArcTanh} \left[\frac{(a-b) \text{Tan} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2 - b^2}} \right] \right) \\
& \text{Log} \left[1 - \frac{\left(b + \text{i} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \text{Tan} \left[\frac{1}{2} (e+f x) \right] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \text{Tan} \left[\frac{1}{2} (e+f x) \right] \right)} \right] + \\
& \text{i} \left(\text{PolyLog} \left[2, \frac{\left(b - \text{i} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \text{Tan} \left[\frac{1}{2} (e+f x) \right] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \text{Tan} \left[\frac{1}{2} (e+f x) \right] \right)} \right] - \right. \\
& \left. \text{PolyLog} \left[2, \frac{\left(b + \text{i} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \text{Tan} \left[\frac{1}{2} (e+f x) \right] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \text{Tan} \left[\frac{1}{2} (e+f x) \right] \right)} \right] \right) \text{Sec} [e + f x]^2 + \\
& \frac{1}{a^2 (a^2 - b^2)^{3/2} f^2 (a + b \text{Sec}[e + f x])^2} 2 b^3 c d (b + a \cos[e + f x])^2 \\
& \left(2 (e + f x) \text{ArcTanh} \left[\frac{(a+b) \text{Cot} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2 - b^2}} \right] - \right. \\
& 2 \left(e + \text{ArcCos} \left[-\frac{b}{a} \right] \right) \text{ArcTanh} \left[\frac{(a-b) \text{Tan} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2 - b^2}} \right] + \\
& \left. \left(\text{ArcCos} \left[-\frac{b}{a} \right] - 2 \text{i} \left(\text{ArcTanh} \left[\frac{(a+b) \text{Cot} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2 - b^2}} \right] - \text{ArcTanh} \left[\frac{(a-b) \text{Tan} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2 - b^2}} \right] \right) \right) \\
& \text{Log} \left[\frac{\sqrt{a^2 - b^2} e^{-\frac{1}{2} i (e+f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \cos[e + f x]}} \right] + \\
& \left(\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \text{i} \left(\text{ArcTanh} \left[\frac{(a+b) \text{Cot} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2 - b^2}} \right] - \text{ArcTanh} \left[\frac{(a-b) \text{Tan} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2 - b^2}} \right] \right) \right) \\
& \text{Log} \left[\frac{\sqrt{a^2 - b^2} e^{\frac{1}{2} i (e+f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \cos[e + f x]}} \right] - \left(\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \text{i} \text{ArcTanh} \left[\frac{(a-b) \text{Tan} \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2 - b^2}} \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Log} \left[1 - \frac{\left(b - \frac{i}{2} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan \left[\frac{1}{2} (e + f x) \right] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \tan \left[\frac{1}{2} (e + f x) \right] \right)} \right] + \\
& \left(-\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \frac{i}{2} \text{ArcTanh} \left[\frac{(a-b) \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2 - b^2}} \right] \right) \\
& \text{Log} \left[1 - \frac{\left(b + \frac{i}{2} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan \left[\frac{1}{2} (e + f x) \right] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \tan \left[\frac{1}{2} (e + f x) \right] \right)} \right] + \\
& \frac{i}{2} \left(\text{PolyLog} \left[2, \frac{\left(b - \frac{i}{2} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan \left[\frac{1}{2} (e + f x) \right] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \tan \left[\frac{1}{2} (e + f x) \right] \right)} \right] - \right. \\
& \left. \text{PolyLog} \left[2, \frac{\left(b + \frac{i}{2} \sqrt{a^2 - b^2} \right) \left(a + b - \sqrt{a^2 - b^2} \tan \left[\frac{1}{2} (e + f x) \right] \right)}{a \left(a + b + \sqrt{a^2 - b^2} \tan \left[\frac{1}{2} (e + f x) \right] \right)} \right] \right) \sec [e + f x]^2 - \\
& \left(2 b d^2 e^{i e} (b + a \cos [e + f x])^2 \left(-2 f x \text{PolyLog} \left[2, -\frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] - \right. \right. \\
& \left. \left. \frac{i}{2} \left(f^2 x^2 \text{Log} \left[1 + \frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] - f^2 x^2 \text{Log} \left[1 + \frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] + \right. \right. \\
& \left. \left. 2 \frac{i}{2} f x \text{PolyLog} \left[2, -\frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] + \right. \right. \\
& \left. \left. 2 \text{PolyLog} \left[3, -\frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] - 2 \text{PolyLog} \left[3, -\frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] \right) \right) \sec [e + f x]^2 \Bigg) / \left((a^2 - b^2) \sqrt{(-a^2 + b^2) e^{2 i e}} f^3 (a + b \sec [e + f x])^2 \right) + \\
& \left(b^3 d^2 e^{i e} (b + a \cos [e + f x])^2 \left(-2 f x \text{PolyLog} \left[2, -\frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] - \right. \right. \\
& \left. \left. \frac{i}{2} \left(f^2 x^2 \text{Log} \left[1 + \frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] - f^2 x^2 \text{Log} \left[1 + \frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] + \right. \right. \\
& \left. \left. 2 \frac{i}{2} f x \text{PolyLog} \left[2, -\frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] + \right. \right. \\
& \left. \left. 2 \text{PolyLog} \left[3, -\frac{a e^{i (2 e+f x)}}{b e^{i e} - \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] - 2 \text{PolyLog} \left[3, -\frac{a e^{i (2 e+f x)}}{b e^{i e} + \sqrt{(-a^2 + b^2) e^{2 i e}}} \right] \right) \right)
\end{aligned}$$

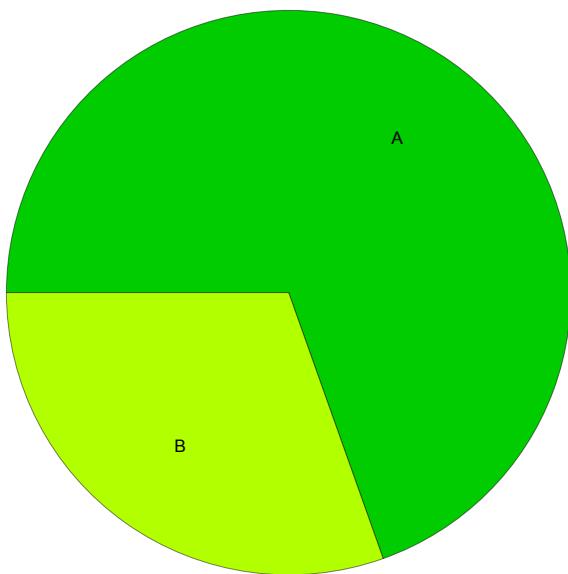
$$\begin{aligned}
& \left. \frac{\sec(e + f x)^2}{\left(a^2 (a^2 - b^2) \sqrt{(-a^2 + b^2) e^{2i e}} f^3 (a + b \sec(e + f x))^2 \right)} - \right. \\
& \left. \left(4 \pm b c^2 \operatorname{ArcTan} \left[\frac{-i a \sin(e) - i (-b + a \cos(e)) \tan \left[\frac{f x}{2} \right]}{\sqrt{-b^2 + a^2 \cos(e)^2 + a^2 \sin(e)^2}} \right] (b + a \cos(e + f x))^2 \sec(e + f x)^2 \right) \right. \\
& \left. \left((a^2 - b^2) f (a + b \sec(e + f x))^2 \sqrt{-b^2 + a^2 \cos(e)^2 + a^2 \sin(e)^2} \right) + \right. \\
& \left. \left(2 \pm b^3 c^2 \operatorname{ArcTan} \left[\frac{-i a \sin(e) - i (-b + a \cos(e)) \tan \left[\frac{f x}{2} \right]}{\sqrt{-b^2 + a^2 \cos(e)^2 + a^2 \sin(e)^2}} \right] (b + a \cos(e + f x))^2 \sec(e + f x)^2 \right) \right. \\
& \left. \left(a^2 (a^2 - b^2) f (a + b \sec(e + f x))^2 \sqrt{-b^2 + a^2 \cos(e)^2 + a^2 \sin(e)^2} \right) + \right. \\
& \left. \left(2 b^2 c d (b + a \cos(e + f x))^2 \sec(e) \sec(e + f x)^2 \right. \right. \\
& \left. \left. \left(a \cos(e) \log[b + a \cos(e) \cos(f x)] - a \sin(e) \sin(f x) \right) + \right. \right. \\
& \left. \left. \left. 2 \pm a b \operatorname{ArcTan} \left[\frac{-i a \sin(e) - i (-b + a \cos(e)) \tan \left[\frac{f x}{2} \right]}{\sqrt{-b^2 + a^2 \cos(e)^2 + a^2 \sin(e)^2}} \right] \sin(e) \right) \right) \right. \\
& \left. \left. \left. a f x \sin(e) - \frac{1}{\sqrt{-b^2 + a^2 \cos(e)^2 + a^2 \sin(e)^2}} \right) \right) \right. \\
& \left. \left. \left(a (a^2 - b^2) f^2 (a + b \sec(e + f x))^2 (a^2 \cos(e)^2 + a^2 \sin(e)^2) \right) - \right. \right. \\
& \left. \left. \frac{1}{a (a^2 - b^2) f (a + b \sec(e + f x))^2} \right. \right. \\
& \left. \left. 2 b^2 d^2 (b + a \cos(e + f x))^2 \sec(e) \sec(e + f x)^2 \right. \right. \\
& \left. \left. \left(\frac{x^2 \sin(e)}{2 a} - \frac{1}{a f} x \left(\cos(e) \log[b + a \cos(e + f x)] + f x \sin(e) \right. \right. \right. \right. \\
& \left. \left. \left. \left. + b \operatorname{ArcTan} \left[\left((\pm \cos(e) + \sin(e)) \left(a \sin(e) + (-b + a \cos(e)) \tan \left[\frac{f x}{2} \right] \right) \right) \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left(\sqrt{a^2 - b^2} \sqrt{(\cos(e) - \pm \sin(e))^2} \right) \right] (2 \sin(e)^2 + \pm \sin(2e)) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{a^2 - b^2} \sqrt{(\cos[e] - i \sin[e])^2} \right) + \frac{1}{a f} \left(\frac{(\epsilon + f x) \cos[e] \log[b + a \cos[e + f x]]}{f} + \right. \\
& \left. \frac{1}{f} a \cos[e] \left(-\frac{(\epsilon + f x) \log[b + a \cos[e + f x]]}{a} + \frac{1}{a} \left(\frac{1}{2} i (\epsilon + f x)^2 - 4 i \arcsin\left[\frac{\sqrt{\frac{a+b}{a}}}{\sqrt{2}}\right] \right. \right. \right. \\
& \left. \left. \left. \arctan\left[\frac{(-a+b) \tan\left[\frac{1}{2}(\epsilon + f x)\right]}{\sqrt{-a^2 + b^2}}\right] - \left(\epsilon + f x + 2 \arcsin\left[\frac{\sqrt{\frac{a+b}{a}}}{\sqrt{2}}\right] \right) \right. \right. \\
& \left. \left. \left. \log\left[1 + \frac{(b - \sqrt{-a^2 + b^2}) e^{i(\epsilon + f x)}}{a}\right] - \left(\epsilon + f x - 2 \arcsin\left[\frac{\sqrt{\frac{a+b}{a}}}{\sqrt{2}}\right] \right) \right. \right. \\
& \left. \left. \left. \log\left[1 + \frac{(b + \sqrt{-a^2 + b^2}) e^{i(\epsilon + f x)}}{a}\right] + (\epsilon + f x) \log[b + a \cos[e + f x]] + i \left(\text{PolyLog}[\right. \right. \right. \\
& \left. \left. \left. 2, -\frac{(b - \sqrt{-a^2 + b^2}) e^{i(\epsilon + f x)}}{a} \right] + \text{PolyLog}[2, -\frac{(b + \sqrt{-a^2 + b^2}) e^{i(\epsilon + f x)}}{a}] \right) \right) + \right. \\
& \left. \left(b \times \arctan\left[\left((\pm \cos[e] + \sin[e]) \left(a \sin[e] + (-b + a \cos[e]) \tan\left[\frac{f x}{2}\right]\right)\right] \right. \right. \right. \\
& \left. \left. \left. \left(\sqrt{a^2 - b^2} \sqrt{(\cos[e] - i \sin[e])^2} \right) \right) (2 \sin[e]^2 + i \sin[2e]) \right) \right. \right. \\
& \left. \left. \left. \left(\sqrt{a^2 - b^2} \sqrt{(\cos[e] - i \sin[e])^2} \right) - \frac{1}{2 \sqrt{a^2 - b^2} f (\cos[e] - i \sin[e])^2} \right. \right. \\
& b \left(2 (\epsilon + f x) \operatorname{ArcTanh}\left[\frac{(a+b) \cot\left[\frac{1}{2}(\epsilon + f x)\right]}{\sqrt{a^2 - b^2}}\right] - 2 \left(\epsilon + \operatorname{ArcCos}\left[-\frac{b}{a}\right]\right) \operatorname{ArcTanh}\left[\right. \right. \\
& \left. \left. \left. \frac{(a-b) \tan\left[\frac{1}{2}(\epsilon + f x)\right]}{\sqrt{a^2 - b^2}}\right] + \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 i \operatorname{ArcTanh}\left[\frac{(a+b) \cot\left[\frac{1}{2}(\epsilon + f x)\right]}{\sqrt{a^2 - b^2}}\right]\right. \right. \\
& \left. \left. \left. 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \tan\left[\frac{1}{2}(\epsilon + f x)\right]}{\sqrt{a^2 - b^2}}\right]\right) \operatorname{Log}\left[\frac{\sqrt{a^2 - b^2} e^{-\frac{1}{2} i(\epsilon + f x)}}{\sqrt{2} \sqrt{a} \sqrt{b + a \cos[e + f x]}}\right] + \right)
\end{aligned}$$

$$\begin{aligned}
& 2 \left(e + \text{ArcCos} \left[-\frac{b}{a} \right] \right) \text{ArcTanh} \left[\frac{(a-b) \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2-b^2}} \right] + \\
& \left(\text{ArcCos} \left[-\frac{b}{a} \right] - 2 \operatorname{Im} \left(\text{ArcTanh} \left[\frac{(a+b) \cot \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2-b^2}} \right] - \text{ArcTanh} \left[\frac{(a-b) \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2-b^2}} \right] \right) \right) \\
& \text{Log} \left[\frac{\sqrt{a^2-b^2} e^{-\frac{1}{2} \operatorname{Im} (e+f x)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \cos [e+f x]}} \right] + \\
& \left(\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \operatorname{Im} \left(\text{ArcTanh} \left[\frac{(a+b) \cot \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2-b^2}} \right] - \text{ArcTanh} \left[\frac{(a-b) \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2-b^2}} \right] \right) \right) \\
& \text{Log} \left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2} \operatorname{Im} (e+f x)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \cos [e+f x]}} \right] - \\
& \left(\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \operatorname{Im} \text{ArcTanh} \left[\frac{(a-b) \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2-b^2}} \right] \right) \\
& \text{Log} \left[1 - \frac{\left(b - \operatorname{Im} \sqrt{a^2-b^2} \right) \left(a+b - \sqrt{a^2-b^2} \tan \left[\frac{1}{2} (e+f x) \right] \right)}{a \left(a+b + \sqrt{a^2-b^2} \tan \left[\frac{1}{2} (e+f x) \right] \right)} \right] + \\
& \left(-\text{ArcCos} \left[-\frac{b}{a} \right] + 2 \operatorname{Im} \text{ArcTanh} \left[\frac{(a-b) \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{a^2-b^2}} \right] \right) \\
& \text{Log} \left[1 - \frac{\left(b + \operatorname{Im} \sqrt{a^2-b^2} \right) \left(a+b - \sqrt{a^2-b^2} \tan \left[\frac{1}{2} (e+f x) \right] \right)}{a \left(a+b + \sqrt{a^2-b^2} \tan \left[\frac{1}{2} (e+f x) \right] \right)} \right] + \\
& \operatorname{Im} \left(\text{PolyLog} \left[2, \frac{\left(b - \operatorname{Im} \sqrt{a^2-b^2} \right) \left(a+b - \sqrt{a^2-b^2} \tan \left[\frac{1}{2} (e+f x) \right] \right)}{a \left(a+b + \sqrt{a^2-b^2} \tan \left[\frac{1}{2} (e+f x) \right] \right)} \right] - \right. \\
& \left. \text{PolyLog} \left[2, \frac{\left(b + \operatorname{Im} \sqrt{a^2-b^2} \right) \left(a+b - \sqrt{a^2-b^2} \tan \left[\frac{1}{2} (e+f x) \right] \right)}{a \left(a+b + \sqrt{a^2-b^2} \tan \left[\frac{1}{2} (e+f x) \right] \right)} \right] \right) \text{Sec} [e+f x]^2 \tan [e] + \\
& \left(4 \operatorname{Im} b^3 c d \text{ArcTan} \left[\frac{-\operatorname{Im} a \sin [e] - \operatorname{Im} (-b+a \cos [e]) \tan \left[\frac{f x}{2} \right]}{\sqrt{-b^2+a^2 \cos [e]^2+a^2 \sin [e]^2}} \right] \right. \\
& \left. \left(b+a \cos [e+f x] \right)^2 \right. \\
& \left. \text{Sec} [e+f x]^2 \right. \\
& \left. \tan [e] \right) / \\
& \left(a^2 (a^2-b^2) f^2 (a+b \sec [e+f x])^2 \sqrt{-b^2+a^2 \cos [e]^2+a^2 \sin [e]^2} \right)
\end{aligned}$$

Summary of Integration Test Results

46 integration problems



- A - 32 optimal antiderivatives
- B - 14 more than twice size of optimal antiderivatives
- C - 0 unnecessarily complex antiderivatives
- D - 0 unable to integrate problems
- E - 0 integration timeouts